ENGLISH

Bluebonnet Learning™
Secondary Mathematics
EDITION 1

Grade 6

Family Guides

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Acknowledgment

Thank you to all the Texas educators and stakeholders who supported the review process and provided feedback. These materials are the result of the work of numerous individuals, and we are deeply grateful for their contributions.

Notice

These learning resources have been built for Texas students, aligned to the Texas Essential Knowledge and Skills, and are made available pursuant to Chapter 31, Subchapter B-1 of the Texas Education Code.

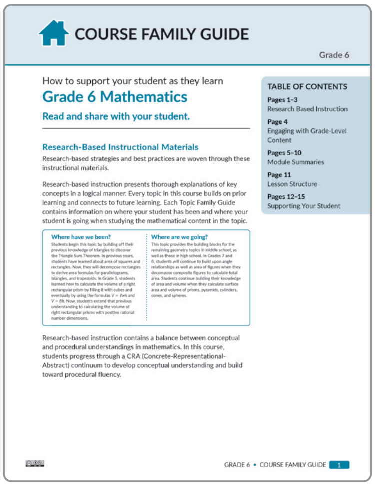
If you have further product questions or to report an error, please email openeducationresources@tea.texas.gov.

Dear Family,

We recognize that learning outside of the classroom is crucial to your student's success at school. This letter serves as an introduction to the resources designed to assist you as you talk to your student about what they are learning. Resources available to you include:

- The Course Family Guide
- Topic Family Guides
- Topic Summaries
- The Mathematics Glossary

Course Family Guide


COURSE FAMILY GUIDE

Grade 6

How to support your student as they learn

Grade 6 Mathematics

Read and share with your student.

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Research-Based Instructional Materials

Research-based strategies and best practices are woven through these instructional materials.

Research-based instruction presents thorough explanations of key concepts in a logical manner. Every topic in this course builds on prior learning and connects to future learning. Each Topic Family Guide contains information on where your student has been and where your student is going when studying the mathematical content in the topic.

Where have we been?
Students begin this topic by building off their previous knowledge of triangles to discover the Triangle Sum Theorem. In previous years, students have learned about area of squares and rectangles. Now, they will decompose rectangles to derive area formulas for parallelograms, triangles, and trapezoids. In Grade 5, students learned how to calculate the volume of a right rectangular prism by filling it with cubes and eventually by using the formulas $V = \ell wh$ and $V = Bh$. Now, students extend that previous understanding to calculating the volume of right rectangular prisms with positive rational number dimensions.

Where are we going?
This topic provides the building blocks for the remaining geometry topics in middle school, as well as those in high school. In Grades 7 and 8, students will continue to build upon angle relationships as well as area of figures when they decompose composite figures to calculate total area. Students continue building their knowledge of area and volume when they calculate surface area and volume of prisms, pyramids, cylinders, cones, and spheres.

Research-based instruction contains a balance between conceptual and procedural understandings in mathematics. In this course, students progress through a CRA (Concrete-Representational-Abstract) continuum to develop conceptual understanding and build toward procedural fluency.

GRADE 6 • COURSE FAMILY GUIDE 1

Following this letter, there is the Course Family Guide that will walk you through the research-based instructional approach, how the course is structured, how to bust math myths, using Talking Points from the Topic Family Guide, and using the TEKS mathematical process standards to initiate discussions.

Research and classroom experience guided course development, with the foundation being a scientific understanding of how people learn and a real-world understanding of how to apply that science to mathematics instructional materials. The instructional design elements presented in the Course Family Guide incorporate research-based strategies to develop conceptual understanding and creative problem solvers.

The Course Family Guide provides an overview of the structure of the course. The course consists of both a Learning Together component and a Learning Individually component. The teacher facilitates a collaborative learning experience during the Learning Together Days and uses data to target specific skills on the Learning Individually Days.

Next, the Course Family Guide includes Module Overviews of each module in the course, which include a detailed summary of what your student will be learning in each topic within the module. Below the topic summaries are facts and information that connect the concepts to the real world. Read and discuss the information below the topic summaries with your student and continue to come back to these pages as your student moves from one topic to the next within each module.

The Course Family Guide also highlights the lesson structure. Each lesson is structured the same way and includes four parts: Objectives & Essential Question, Getting Started, Activities, and the Talk the Talk.

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Topic Family Guide

Each course is organized into modules. Each module consists of topics with corresponding Topic Family Guides. These guides all have the same structure. This consistency will allow you and your student to understand how to reference the content of each topic.

MYTH

Asking questions means you don't understand.

It is universally true that, for any given body of knowledge, there are levels to understanding. For example, you might understand the rules of baseball and follow a game without trouble. However, there is probably more to the game that you can learn. For example, do you know the 23 ways to get on first base, including the one where the batter strikes out?

Questions don't always indicate a lack of understanding. Instead, they might allow you to learn even more about a subject that you already understand. Asking questions may also give you an opportunity to ensure that you understand a topic correctly. Finally, questions are extremely important to ask yourself. For example, ***everyone*** should be in the habit of asking themselves, "Does that make sense? How would I explain it to a friend?"

#mathmythbusted

The Topic Family Guide begins with an overview of the content in the topic. This introduction includes a brief explanation of what your student will learn in the topic, the prior knowledge they will use to help them understand this topic, and a connection to future learning.

The next section of the Topic Family Guide is the Talking Points section. The Talking Points section provides questions you can ask as your student works through the math of the topic.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is becoming fluent with fraction operations and gaining experience with the area of two-dimensional shapes and the volume of right rectangular prisms.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? Why or why not?
- Is there anything you don't understand? How can you use today's lesson to help?

Next, the Topic Family Guide lists all the new key terms of the topic and details some of the math strategies students will learn in the topic. Finally, each Topic Family Guide contains a Math Myth. Busting these Math Myths helps to build confidence and explain how math is accessible to everyone.

Topic Summary

A Topic Summary is provided for students at the end of each topic. The Topic Summary lists all new key terms of the topic and provides a summary of each lesson. Each lesson summary defines new key terms and reviews key concepts, strategies, and/or Worked Examples. The Topic Summary provides an opportunity for you and your student to discuss the key concepts from each lesson, review the examples, and do the math together.

LESSON 1

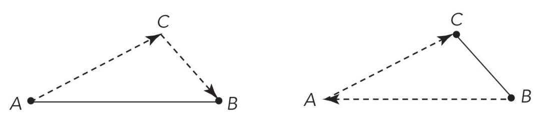
Constructing Triangles Given Sides

Triangles are congruent when all of their corresponding angle measures and corresponding side lengths are the same. When given information can be used to construct congruent triangles, the information is said to define a unique triangle.

The **Triangle Inequality Theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AC + CB > AB$

$BA + AC > BC$

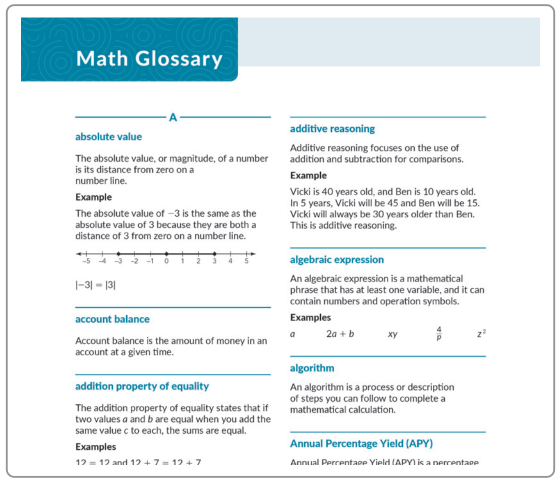


When given two line segments, it is possible to construct an infinite number of triangles. When given three line segments, it is possible to either construct a unique triangle or no triangle.

Evidence of the TEKS mathematical process standards are present in the Topic Summaries. Each lesson within the topic highlights one or more of the TEKS mathematical process standards. These processes will help your student develop effective communication and collaboration skills that are essential for becoming a successful learner. Discuss with your student the “I can” statements associated with each of the TEKS mathematical process standards to help them develop their mathematical learning and understanding. The “I can” statements for each of the TEKS mathematical process standards are included in the Course Family Guide. With your help, your student can develop the habits of a productive mathematical thinker.

Math Glossary

The Math Glossary for each course is a tool for your student to utilize and reference during their learning. Along with the definition of a term, the glossary provides examples to help further their understanding.



Math Glossary

A

absolute value

The absolute value, or magnitude, of a number is its distance from zero on a number line.

Example

The absolute value of -3 is the same as the absolute value of 3 because they are both a distance of 3 from zero on a number line.

$|-3| = |3|$

account balance

Account balance is the amount of money in an account at a given time.

addition property of equality

The addition property of equality states that if two values a and b are equal when you add the same value c to each, the sums are equal.

Examples

$12 = 12$ and $12 + 7 = 12 + 7$

additive reasoning

Additive reasoning focuses on the use of addition and subtraction for comparisons.

Example

Vicki is 40 years old, and Ben is 10 years old. In 5 years, Vicki will be 45 and Ben will be 15. Vicki will always be 30 years older than Ben. This is additive reasoning.

algebraic expression

An algebraic expression is a mathematical phrase that has at least one variable, and it can contain numbers and operation symbols.

Examples

a $2a + b$ xy $\frac{4}{p}$ z^2

algorithm

An algorithm is a process or description of steps you can follow to complete a mathematical calculation.

Annual Percentage Yield (APY)

Annual Percentage Yield (APY) is a percentage

We all have the same goal for your student: to become a successful problem solver and use mathematics efficiently and effectively in daily life. Encourage them to use the mathematics they already know when seeing new concepts and communicate their thinking while providing a critical ear to the thinking of others.

Thank you for supporting your student.

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COURSE FAMILY GUIDE

Grade 6

How to support your student as they learn

Grade 6 Mathematics

Read and share with your student.

TABLE OF CONTENTS

Research-Based Instruction

Research-based strategies and best practices are woven through these instructional materials.

Thorough explanation of key concepts in a logical manner. Every topic in this course builds on prior learning and connects to future learning. Each Topic Family Guide contains information on where your student has been and where your student is going when studying the mathematical content in the topic.

Where have we been?

Students begin this topic by building on their previous knowledge of triangles to discover the Triangle Sum theorem. In previous courses, students have learned about area of squares and rectangles. Now, they will decompose rectangles to derive area formulas for parallelograms, triangles, and trapezoids. In Grade 5, students learned how to calculate the volume of a right rectangular prism by filling it with cubes and eventually by using the formulas $V = \ell wh$ and $V = Bh$. Now, students extend that previous understanding to calculating the volume of right rectangular prisms with positive rational number dimensions.

Where are we going?

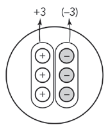
This topic provides the building blocks for the remaining geometry topics in future courses. In Grades 7 and 8, students will continue to build upon angle relationships as well as area of figures when they decompose composite figures to calculate total area. Students continue building their knowledge of area and volume when they calculate surface area and volume of prisms, pyramids, cylinders, cones, and spheres.

The instructional materials balance conceptual and procedural understandings. In this course, students progress through a Concrete-Representational-Abstract (CRA) continuum to develop conceptual understanding and build toward procedural fluency.

Concrete	Representational	Abstract
Students interact with a Worked Example as they utilize two-color counters to add integers.	Students draw models representing the addition of number lines.	Students add integers without the use of counters.

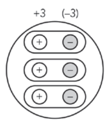
WORKED EXAMPLE

You can model the expression $3 + (-3)$ in different ways using two-color counters:



Three positive charges and three negative charges have no charge.

$3 + (-3) = 0$



Each positive charge is paired with a negative charge.

Each pair of positive and negative charges has no charge.

$3 + (-3) = 0$

Draw a model for each and then complete the number sentence.

a. $-9 + (-4) =$ ______

b. $-4 + (-5) + 4 =$ ______

c. $3 + 6 + (-4) =$ ______

$-58 + 24$	$-35 + (-15)$
$-33 + (-12)$	$-48 + 60$
$26 + (-13)$	$67 + 119$
$-105 + 25$	$153 + (-37)$
$21 + (-56)$	$18 + (-17)$

Support is provided to students as they persevere in problem solving. These instructional materials include a problem-solving model, which includes questions your student can ask when productively engaging in real-world and mathematical problems. Prompts will encourage your student to use the problem-solving model throughout the course.

These instructional materials include features that support learners. Worked Examples throughout the product provide explicit instruction and provide a model your student can continually reference.

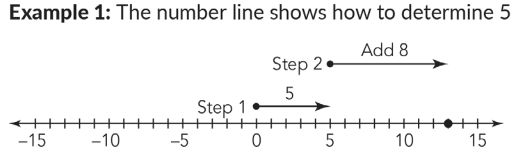
When you see a Worked Example:

- Take your time to read through it.
- Question your own understanding.
- Think about the connection between steps.

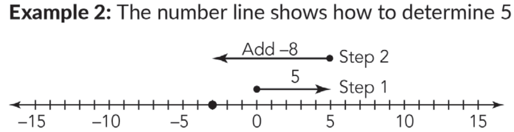
WORKED EXAMPLE

A number line can be used to model integer addition.
When adding a positive integer, move to the right on a number line.
When adding a negative integer, move to the left on a number line.

Example 1: The number line shows how to determine $5 + 8$.



Example 2: The number line shows how to determine $5 + (-8)$.



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Thumbs Up, Thumbs Down, and Who's Correct questions address your student's common misconceptions and provide opportunities for peer work analysis.

Thumbs Up Thumbs Down

When you see a Thumbs Up icon:

- Take your time to read through the correct solution.
- Think about the connection between steps.

Ask Yourself

- Why is this method correct?
- Have I used this method before?

> Clover
>
> I can determine the unknown number in $8 + 4 = \_\_ + 5$ by rewriting the expression on the left. I can take 1 from 8 and give it to the 4 and keep the value of the expression the same.
>
> $(8 - 1) + (4 + 1) = \_\_ + 5$
>
> $7 + 5 = \_\_ + 5$
>
> Therefore, the unknown number is 7.

When you see a Thumbs Down icon:

- Take your time to read through the incorrect solution.
- Think about what error was made.

Ask Yourself

- Where is the error?
- Why is it an error?
- How can I correct it?

> Rylee
>
> The equals sign tells me to perform the operation on the left in the equation $8 + 4 = \_\_ + 5$.
>
> $8 + 4 = 12 + 5$
>
> $12 + 5 = 17$
>
> Therefore, the unknown number is 17.

Who's Correct

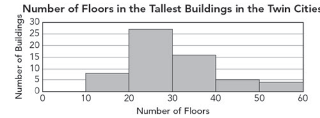
When you see a Who's Correct icon:

- Take your time to read through the situation.
- Question the strategy or reason given.
- Determine if correct or incorrect.

Ask Yourself

- Does the reasoning make sense?
- If the reasoning makes sense, what is the justification?
- If the reasoning does not make sense, what error was made?

Number of Floors in the Tallest Buildings in the Twin Cities



Bella says, "There are 5 buildings represented in the histogram since there are 5 bars." Do you agree or disagree with Bella's statement? If you do not agree with Bella, estimate how many buildings are represented in the histogram.

Targeted Skills Practice provides support to your student as they work to gain proficiency of the course material. Spaced Practice provides a spaced retrieval of key concepts to your student. Extension opportunities provide challenges to accelerate your student's learning.

Skills Practice TOPIC 2 Operating with Integers

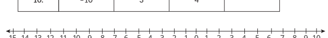
Name ____________________ Date __________

I. Using Models to Understand Integer Addition

Topic Practice

A. Use the number line to determine the ending position by adding and subtracting the indicated steps from each starting position.

	Starting Position	Steps Backward	Steps Forward	Ending Position
1.	+1	3	6	
2.	+5	6	2	
3.	+3	6	4	
4.	0	9	12	
5.	−5	2	7	
6.	+1	8	9	
7.	−5	3	5	
8.	−1	9	6	
9.	+8	4	2	
10.	−10	3	4	



TOPIC 2 Operating with Integers

Extension

Draw a model to represent the addition problem $-3\frac{1}{2} + \left(-1\frac{1}{4}\right)$. Then, determine the solution.

Spaced Practice

Write an absolute value statement and an integer to represent each situation.

1. The temperature went from 45 degrees Fahrenheit to 10 degrees Fahrenheit.
2. The value of a stock went from \$500 to \$250 last year.
3. The water level dropped from 10 feet to 2 feet.

Each lesson features one or more ELPS (English Language Proficiency Standards) and provides the teacher with implementation strategies incorporating best practices for supporting language acquisition. In addition, students are provided with cognates for New Key Terms in the Topic Summaries and Topic Family Guides.

NEW KEY TERMS

- Triangle Inequality Theorem
- Triangle Sum Theorem [Teorema de la suma del triángulo]
- parallelogram [paralelogramo]
- variable [variable]
- straightedge
- trapezoid [trapecio]
- geometric solid
- polyhedron [poliedro]
- face
- edge
- vertex [vértice]
- right rectangular prism [prisma rectangular recto]
- cube [cubo]
- volume [volumen]

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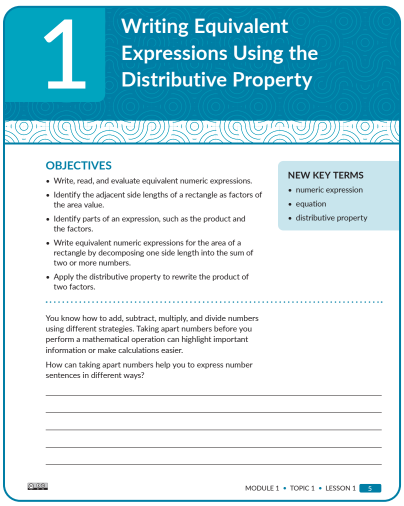
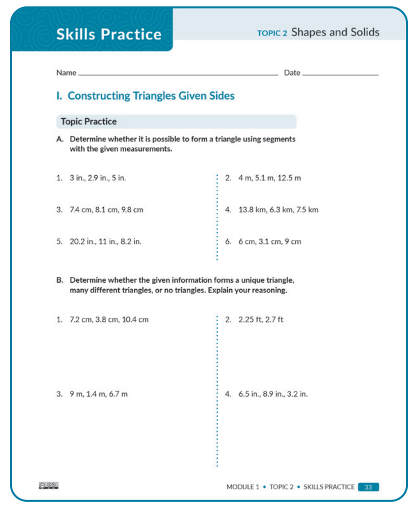
Engaging with Grade-Level Content

Your student will engage with grade-level content in multiple ways with the support of the teacher.

Learning Together	Learning Individually
The teacher facilitates active learning of lessons so that students feel confident in sharing ideas, listening to each other, and learning together. Students become creators of their mathematical knowledge. 	Skills Practice provides students the opportunity to engage in additional skill building that aligns to each Learning Together lesson. On Learning Individual Days, teachers use Skills Practice to target discrete skills that may require additional practice to achieve proficiency. 

At the end of each topic, your student will take an assessment aligned to the standards covered in the topic. This assessment consists of multiple-choice, multiselect, and open-ended questions designed for your student to demonstrate learning. Each assessment also includes a scoring guide for teachers to ensure consistent scoring. The scoring guide includes ways to support or challenge your student based on their responses to the questions on the assessment. The purpose of the assessment is for the teacher and student to reflect on the learning. Teachers will use your student's assessment results to target individual skills your student needs for proficiency or to accelerate and challenge your student.

Response to Student Performance		
TEKS*	Question(s)	Recommendations
6.2C	1, 5	To support students: • Review how to locate points on a number line. • Use Skills Practice Set I.A for additional practice. • Review Lesson 1 Assignment Practice Questions 1, 2, and 4.
6.2D	2, 6	To support students: • Review ordering rational numbers. • Use Skills Practice Sets I.B and I.C for additional practice. • Review Lesson 1 Assignment Practice Questions 3 and 5. To challenge students: • Extend student knowledge with the Skills Practice Extension Set I.
6.3E	3, 7	To support students: • Review multiplication of positive rational numbers. • Use Skills Practice Sets II.A, II.B, and II.C for additional practice. • Review Lesson 2 Assignment Practice Questions 1 and 2.
	9, 10	To support students: • Review division of positive rational numbers. • Use Skills Practice Sets III.A, III.B, III.C, and III.D for additional practice. • Review Lesson 3 Assignment Practice Questions 1–3.

MODULE 1 Composing and Decomposing

In this module, your student will learn more about numbers and shapes and their relationships. There are three topics in this module: *Factors and Multiples*, *Shapes and Solids*, and *Decimals*. Your student will use what they already know about area, number properties, and volume in this module.

TOPIC 1 Factors and Multiples	TOPIC 2 Shapes and Solids	TOPIC 3 Decimals
Your student will study the relationship between numbers and area, review fraction multiplication, and use the inverse relationship between multiplication and division to understand fraction by fraction division.	Your student will study the relationships of angles and side lengths of triangles, as well as the area of triangles, parallelograms, and trapezoids, by decomposing and composing parts of shapes.	Your student will plot, compare, and order decimals on a number line and understand decimal multiplication and division.
What in the world?  Dividing fractions is commonly used when cooking and baking. How many cups of chopped pecans would you need for half the serving size? [$\frac{1}{6}$ *cup of chopped pecans*]	Did you know?  A composite figure is a figure made up of more than one simple geometric figure. The house shown above is a composite figure. Can you name 4 different shapes in the image? [*rectangle, triangle, square, trapezoid*]	Did you know?  You can compare decimals to solve real-world problems. Which runner ran the fastest? [*Vonetta*]

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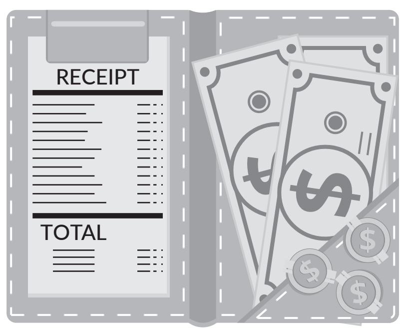
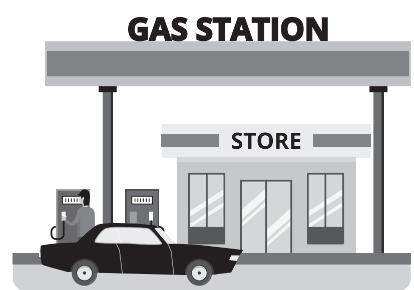
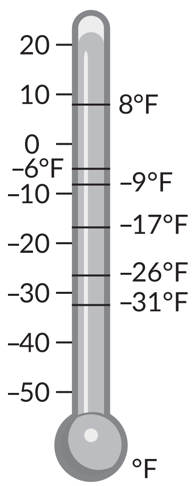
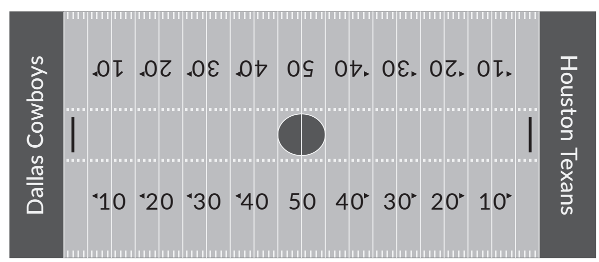
MODULE 2 Relating Quantities

In this module, your student will learn more about ratio and proportional relationships. There are three topics in this module: *Ratios*, *Percents*, and *Unit Rates and Conversions*. Your student will use what they already know about equivalent fractions in this module.

TOPIC 1 Ratios and Rates	TOPIC 2 Percents	TOPIC 3 Unit Rates and Conversions
Your student will use their multiplicative reasoning to establish ratio and rate reasoning.	Your student will analyze percents as a special type of ratio, a rate per 100.	Your student will develop an understanding of unit rates, a special type of ratio, and conversion rates, a special type of unit rate.
Did you know?  When the creamer hits the coffee, the math is magical. With each cup, you create a **ratio** of creamer to coffee. A ratio is a comparison of two quantities that uses division. You can compare the ratio of creamer to coffee without measuring or counting quantities. When you reason like this, it is called *qualitative reasoning*. Which cup contains the coffee with the strongest coffee flavor?	Did you know?  Deciding how much tip to leave a server at a restaurant is one way that percents are used in the real world. To leave a 15% tip, you can easily calculate 10% of any number and then calculate half of that, which is equal to 5%. You can add those two percent values together to get a sum of 15%. What is 15% of $60? [*$9*]	Did you know?  When you decide to bypass one gas station for the one down the road because their gas is a few cents cheaper, you are comparing unit rates. A unit rate is a comparison of two measurements in which the denominator has a value of one unit. Which option is the better buy? Gas station #1: $3.49 per gallon Gas station #2: $3.54 per gallon [*Gas station #1*]

MODULE 3 Moving Beyond Positive Quantities

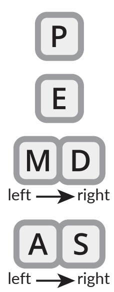
In this module, your student will learn more about positive and negative numbers. There are two topics in this module: *Signed Numbers and the Four Quadrants* and *Operating with Integers*. Your student will use what they already know about adding, subtracting, multiplying, and dividing whole numbers in this module.

TOPIC 1 Signed Numbers and the Four Quadrants	TOPIC 2 Operating with Integers
Your student will use a number line to reason about negative numbers and absolute value. Students explore the four-quadrant coordinate plane.	Your student will use physical motion, number lines, and two-color counters to understand adding and subtracting integers. Students model the product of signed numbers using number lines and two-color counters.
Did you know that? You can use the thermometer to compare signed numbers.  –26°F > –31°F –6°F > –17°F 8°F > –9°F	Did you know that? Each time you watch a game of American football, you are watching Operating with Integers in action. The field is a number line, and the players are modeling integer addition and subtraction when they run up and down the field.  Picture this: A Texan has the ball at his own 30 yardline. How far does he have to go to reach the Cowboys' end zone? Write an equation that represents the situation. [|–*20 yards*| + *50 yards* = *70 yards*]

MODULE 4 Determining Unknown Quantities

In this module, your student will learn more about variables and equations. There are four topics in this module: *Expressions*, *Equations and Inequalities*, *Graphing Quantitative Relationships*, and *Financial Literacy: Accounts, Credit, and Careers*. Your student will use what they already know about expressions, patterns, and numeric operations in this module.

<table>
<tr><th>TOPIC 1
Expressions</th><th>TOPIC 2
Equations and Inequalities</th><th>TOPIC 3
Graphing Quantitative Relationships</th><th>TOPIC 4
Financial Literacy: Accounts, Credit, and Careers</th></tr>
<tr><td>Your student will use the order of operations with numeric expressions to include exponents.</td><td>Your student will learn that the equals sign indicates a relationship between two expressions.</td><td>Your student will analyze relationships between independent and dependent variables, including nonlinear relationships.</td><td>Your student will solve real-world problems focused on financial literacy, including features of checking accounts, debit cards, and credit cards, and learn how education affects career opportunities.</td></tr>
<tr><td>Did you know that?



When an expression includes a combination of parentheses, exponents, multiplication, division, addition, and subtraction, the order in which you perform the operation makes a difference in the answer you get.</td><td>What In The World?

Speed limit signs are designed to communicate a set legal maximum speed that vehicles must travel. Drivers must not exceed the limit that the sign designates.

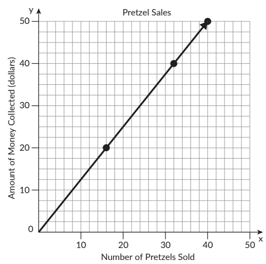
Speed limit signs are examples of **inequalities**.

Speed ≤ 55 mph</td><td>Did you know that?

Every graph tells a story.

a.

b.

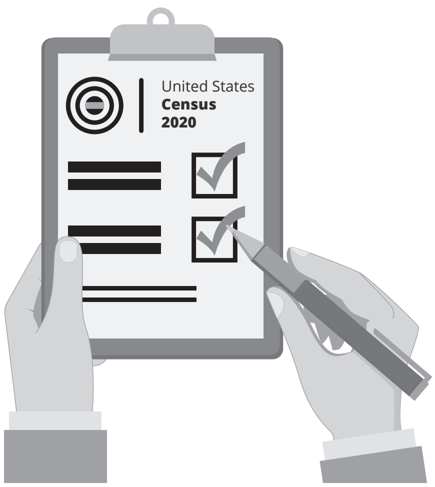


Create a story for each graph.</td><td>Did you know that?

Tuition is a fee paid in order to receive instruction at a school. Tuition may be paid through personal savings, student loans, grants, scholarships, or work-study programs</td></tr>
</table>

MODULE 5 Describing Variability of Quantities

In this module, your student will learn more about the field of statistics, which is the study of data. There are two topics in this module: *The Statistical Process* and *Numerical Summaries of Data*. Your student will use what they already know about representing data with graphs in this module.

TOPIC 1 The Statistical Process	TOPIC 2 Numerical Summaries of Data
You student will learn about the statistical problem-solving process: formulate questions, collect data, analyze data, and interpret the results.	Your student will learn about measures of central tendency and measures of variability.
Did you know?  A census is an official count or survey of a population, typically recording various details of individuals. The data collected by the census tells us who we are and where we are going as a nation, and it helps our communities determine where to build everything from schools to supermarkets and from homes to hospitals.	Did you know?  Colleges use data from schools each year to determine admission. One of the key factors that matter most is a student's grade point average (GPA). Your student's GPA is the sum of all their course grades throughout high school divided by the total number of credits.

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Lesson Structure

Each lesson in the course is laid out in the same way to develop deep understanding. Read through the parts of the lesson to learn more about your student's learning in their math classroom.

Objectives & Essential Question

Each lesson begins with objectives, listed to help students understand the objectives. Also included is an essential statement connecting students' learning with a question to ponder. The question is asked again at the end of each lesson to see how much your student understands.

Getting Started

The Getting Started engages your student in the learning. In the Getting Started, your student uses what they already know about the world, what they've already learned, and their intuition to get them thinking mathematically and prepare them for what's to come in the lesson.

Activities

In the Activities, students develop their mathematical knowledge and build a deep understanding of the math. These activities provide your student with the opportunity to communicate and work with others in their math classroom.

When your student is working through these activities, keep in mind:

- It's not just about answer-getting. Doing the math and talking about it is important.
- Making mistakes is an important part of learning, so take risks.
- There is often more than one way to solve a problem.

Talk the Talk

The Talk the Talk gives your student an opportunity to reflect on the main ideas of the lesson and demonstrate their learning.

Lesson Assignment

The lesson assignment provides your students with practice to develop fluency and build proficiency. The lesson assignment also includes a section to help prepare students for the next lesson.

Key Concepts of the Lesson

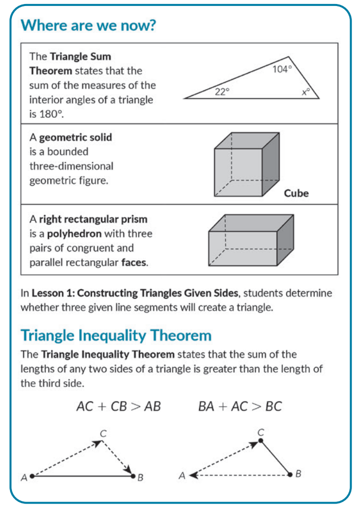
At the end of each topic, the Topic Summary provides a summary of each lesson in the topic. Encourage your student to use these as a tool to review and retrieve the key concepts of a lesson.

Supporting Your Student

The Topic Family Guide

The Topic Family Guide provides an overview of the mathematics of the topic, how that math is connected to what students already know, and how that knowledge will be used in future learning. It provides an example of a math model or strategy, busting of a math myth, talking points to discuss and/or questions to ask your student, and new key terms. You and your student can use the Math Glossary to look up the definitions to the key terms. Encourage your student to reference the new key terms in the Topic Family Guide and/or Math Glossary when completing math tasks.

Learning outside of the classroom is crucial for your student's success. While we don't expect you to be a math teacher, the Topic Family Guide can assist you as you talk to your student about the mathematical content of the course. The hope is that both you and your student will read and benefit from the guides.



Where are we now?

The **Triangle Sum Theorem** states that the sum of the measures of the interior angles of a triangle is 180°.	104°, 22°, x°
A **geometric solid** is a bounded three-dimensional geometric figure.	Cube
A **right rectangular prism** is a **polyhedron** with three pairs of congruent and parallel rectangular **faces**.	

In **Lesson 1: Constructing Triangles Given Sides**, students determine whether three given line segments will create a triangle.

Triangle Inequality Theorem

The **Triangle Inequality Theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$AC + CB > AB$ $\quad$ $BA + AC > BC$

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is becoming fluent with fraction operations and gaining experience with the area of two-dimensional shapes and the volume of right rectangular prisms.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? Why or why not?
- Is there anything you don't understand? How can you use today's lesson to help?

Math Myths

Math myths can lead students and adults to believe that math is too difficult for them, math is an unattainable skill, or that there is only one right way to do mathematics. The Math Myths section in the Topic Family Guides busts these myths and provides research-based explanations of why math is accessible to all students (and adults).

Examples of these myths include:
Myth: Just give me the rule. If I know the rule, then I understand the math.

MYTH

Asking questions means you don't understand.

It is universally true that, for any given body of knowledge, there are levels to understanding. For example, you might understand the rules of baseball and follow a game without trouble. However, there is probably more to the game that you can learn. For example, do you know the 23 ways to get on first base, including the one where the batter strikes out?

Questions don't always indicate a lack of understanding. Instead, they might allow you to learn even more about a subject that you already understand. Asking questions may also give you an opportunity to ensure that you understand a topic correctly. Finally, questions are extremely important to ask yourself. For example, ***everyone*** should be in the habit of asking themselves, "Does that make sense? How would I explain it to a friend?"

#mathmythbusted

Supporting Your Student

Memorize the following rule: *All quars are elos.* Will you remember that rule tomorrow? Nope. Why not? Because it has no meaning. It isn't connected to anything you know. What if we change the rule to: *All squares are parallelograms.* How about now? Can you remember that? Of course you can, because now, it makes sense. Learning does not take place in a vacuum. It must be connected to what you already know. Otherwise, arbitrary rules will be forgotten.

Myth: There is one right way to do math problems.

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. When one road is backed up, you can always take a different route. When you know only one route, you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying, "Well, that's one way to do it. Is there another way? What are the pros and cons?" That way you avoid falling into the trap of thinking there is only one right way because that strategy might not always work or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

TEKS Mathematical Process Standards

Each module will focus on TEKS mathematical process standards that will help your student become a mathematical thinker. The TEKS mathematical process standards are listed below. Discuss with your student the "I can" statements below the standards to help them develop their mathematical learning and understanding. With your help, your student can become a productive mathematical thinker.

Apply mathematics to problems arising in everyday life, society, and the workplace.

I can:

- use the mathematics that I learn to solve real-world problems.
- interpret mathematical results in the contexts of a variety of problem situations.

Supporting Your Student

Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying a solution, and evaluating the problem-solving process and reasonableness of the solution.

I can:

- explain what a problem “means” in my own words.
- create a plan and change it when necessary.
- ask useful questions to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques including mental math, estimation, and number sense as appropriate, to solve problems.

I can:

- use a variety of different tools that I have to solve problems.
- recognize when a tool that I have to solve problems might be helpful and when it has limitations.
- look for efficient methods to solve problems.
- estimate before I begin calculations to inform my reasoning.

Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language, as appropriate.

I can:

- explain what a problem “means” in my own words.
- create a plan and change it if necessary.
- ask useful questions when trying to understand the problem.
- explain my reasoning and defend my solution.
- reflect on whether my results make sense.

Create and use representations to organize, record, and communicate mathematical ideas.

I can:

- consider the units of measure involved in a problem.
- label diagrams and figures appropriately to clarify the meaning of different representations.
- create an understandable representation of a problem situation.

Analyze mathematical relationships to connect and communicate mathematical ideas.

I can:

- identify important relationships in a problem situation.
- use what I know to solve new problems.
- analyze and organize information.
- look closely to identify patterns or structure.
- look for general methods and more efficient ways to solve problems.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

I can:

- work carefully and check my work.
- distinguish correct reasoning from reasoning that is flawed.
- use appropriate mathematical vocabulary when I talk with my classmates, my teacher, and others.
- specify the appropriate units of measure when I explain my reasoning.
- calculate accurately and communicate precisely to others.

Reflecting on Learning and Progress

You can support your student by encouraging your student to reflect on the learning process. The instructional resources include a student Topic Self-Reflection for each topic. Encourage your student to accurately and frequently reflect on learning and progress throughout each topic. Talk about the specific concepts in the Topic Self-Reflection with your student and celebrate the progress from the beginning to the end of the topic. Remind your student to refer to the Topic Self-Reflection on Learning Individual Days after targeting specific skills and concepts. You can have your student explain concepts from the self-reflection using the topic summaries or lesson assignments to demonstrate understanding. In addition, encourage your student to reflect after taking an assessment. An Assessment Reflection is available to your student to assist with this process. Encourage your student to consider what went well and how to prepare for the next assessment. Ask your student how you can support them when preparing for the next assessment.

TOPIC 3 SELF-REFLECTION

Name: ____________________

Decimals

When you reflect on what you are learning, you develop your understanding and know when to ask for help.

Reflect on these statements. Place a number in each circle from 1–3, where 1 represents **the skill is new to me**, 2 represents **I am building proficiency of the skill**, and 3 represents **I have demonstrated proficiency of the skill**.

I can demonstrate an understanding of the standards in the *Decimals* topic by:

TOPIC 3: *Decimals*	Beginning of Topic	Middle of Topic	End of Topic
locating the value of a decimal on a number line			
comparing decimals to the thousandths using $>$, $<$, and $=$			
fluently adding, subtracting, multiplying, and dividing multi-digit decimals using the standard algorithm for each operation with accuracy			
dividing multi-digit decimals using the standard algorithm with accuracy			

continued on the next page

Thanks!

Enjoy the fun mathematical adventure that is ahead for you and your student! Remember the supports available to you, and thank you for supporting your student's learning.

MODULE
1

Composing and Decomposing

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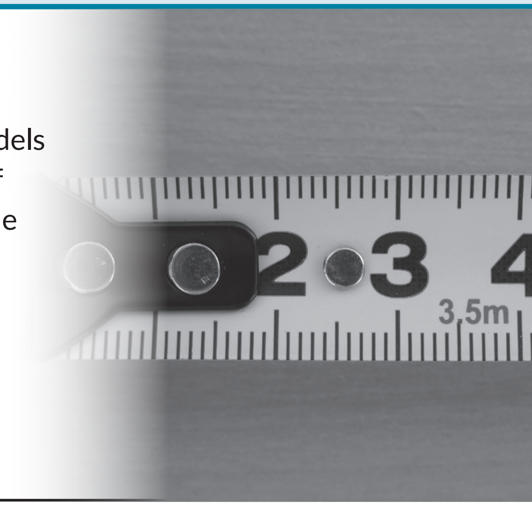
Family Guide

MODULE 1 Composing and Decomposing

Grade 6

TOPIC 1 Factors and Multiples

In this topic, students explore factors and multiples. They use area models to determine the factors of a given number and the common factors of two or more numbers. Students use factor trees to determine the prime factors of a number. They then use models to multiply and divide with fractions before using an algorithm, or step-by-step method.



Where have we been?

Students have used tiling to relate area to multiplication and addition, and they have used informal statements of the properties of operations. Students have also used area models to represent multiplication.

Where are we going?

In the next topic, students will use their knowledge of rectangles and area to develop the area formulas for parallelograms, triangles, and trapezoids. They will continue to build fluency with fractions throughout the course.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Your student is learning to compose and decompose numbers using different techniques. You can further support your student's learning by asking questions about the work they do in class or at home.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

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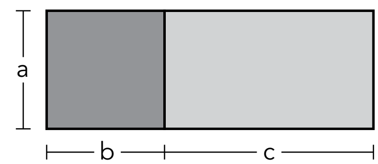
NEW KEY TERMS

- numeric expression [expresión numérica]
- equation [ecuación]
- distributive property [propiedad distributiva]
- base [base]
- power
- exponent [exponente]
- common factor [factor común]
- relatively prime [primos relativos]
- greatest common factor (GCF)
- multiple [múltiple]
- commutative property [propiedad conmutativa]
- least common multiple (LCM) [mínimo común múltiplo]
- unit fraction [fracción unitaria]
- equivalent fraction [fracción equivalente]
- benchmark fractions
- algorithm [algoritmo]
- positive rational number [número racional positivo]
- reciprocal [recíproco]
- multiplicative inverse
- complex fraction [fracción compleja]

Refer to the Math Glossary for definitions of New Key Terms.

Where are we now?

The **distributive property**, when applied for multiplication, states that for any numbers *a*, *b*, and *c*, $a(b + c) = ab + ac$.



The **exponent** of the power is the number of times the base is used as a factor.

exponent

$\downarrow$

$8^4 = 8 \cdot 8 \cdot 8 \cdot 8$

A **multiple** is the product of a given whole number and another whole number.

multiples of 10:

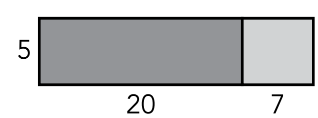
10	20	30	40	50 ...
↑	↑	↑	↑	↑
$10 \cdot 1$	$10 \cdot 2$	$10 \cdot 3$	$10 \cdot 4$	$10 \cdot 5$ ...

In **Lesson 1: Writing Equivalent Expressions Using the Distributive Property**, students will learn more about factors and multiples. They use area models to show the factors of a given number and the **common factors** of two or more numbers.

Area Models

The equation $5 \cdot 27 = 135$ shows that the expression $5 \cdot 27$ is equal to the expression 135. An **equation** is a mathematical sentence that uses an equals sign to show that two or more quantities are the same as one another.

The area model shows the side length of 27 split into two parts.



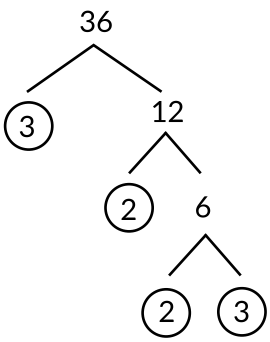
$$5 \cdot 27 = 5(20 + 7)$$

The factors for each region are $(5 \cdot 20) + (5 \cdot 7)$.
The area of each smaller region is $100 + 35$.
The total area is 135.

In **Lesson 2: Identifying Common Factors and Common Multiples**, students use factor trees to determine **prime factorization** of a number.

Factor Trees

A factor tree is a way to organize the prime factors of a number.

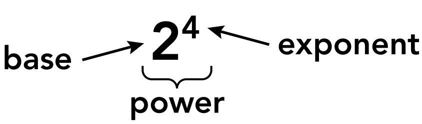


$36 = 2 \cdot 2 \cdot 3 \cdot 3$
$36 = 2^2 \cdot 3^2$

The prime factorization shown has repeated factors. A **power** has two parts: the base and the exponent.

The **base** of a power is the factor multiplied by itself repeatedly, and the **exponent** of the power is the number of times you use the base as a factor.

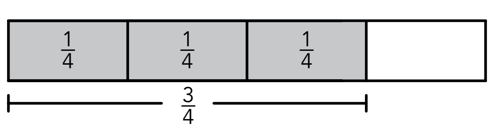
$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$



In **Lesson 6: Fraction by Fraction Division**, students divide fractions using strip diagrams. Students use models of fractions with division, and then use a dividing across strategy.

Strip Diagrams

A strip diagram can show the quotient of two fractions, such as $\frac{3}{4} \div \frac{1}{4}$. The division expression asks, "How many $\frac{1}{4}$ are in $\frac{3}{4}$?"



There are 3 one-fourths in $\frac{3}{4}$, so $\frac{3}{4} \div \frac{1}{4} = 3$.

MYTH

“I don’t have the math gene.”

Let’s be clear about something. There isn’t ***a*** gene that controls the development of mathematical thinking. Instead, there are probably ***hundreds*** of genes that contribute to it.

Some believe mathematical thinking arises from the ability to learn a language. Given the right input from the environment, children learn to speak without formal instruction. They can learn number sense and pattern recognition the same way.

To further nurture your student’s mathematical growth, attend to the learning environment. You can think of it as providing a nutritious mathematical diet that includes: discussing math in the real world, offering encouragement, being available to answer questions, allowing your student to struggle with difficult concepts, and providing space for plenty of practice.

#mathmythbusted

Dividing Fractions

$$\frac{5}{8} \div \frac{3}{4} = \frac{\frac{5}{8}}{\frac{3}{4}}$$

Rewrite the division expression as a **complex fraction**.

$$= \frac{\frac{5}{8}}{\frac{3}{4}} \cdot \frac{\frac{4}{3}}{\frac{4}{3}}$$

Multiply the numerator and denominator by the multiplicative inverse of $\frac{3}{4}$.

$$= \frac{\frac{5}{8}\cdot\frac{4}{3}}{\frac{3}{4}\cdot\frac{4}{3}} = \frac{\frac{5}{8}\cdot\frac{4}{3}}{1}$$

Perform multiplication and rewrite the denominator as 1.

$$= \frac{5}{\cancel{8}_{2}} \cdot \frac{\cancel{4}^{1}}{3} = \frac{5}{6}$$

Family Guide

MODULE 1 Composing and Decomposing

Grade 6

TOPIC 2 Shapes and Solids

In this topic, students determine whether three given line segments will construct a triangle. They use hands-on tools to make and justify conjectures about the sum of the interior angles of a triangle and the relationship between triangle side and angle measures. From their knowledge of rectangles and area, students also develop the formula for the area of parallelograms, triangles, and trapezoids. Students build on their prior knowledge of volume of cubes and rectangular prisms with whole length dimensions to calculating volume of right rectangular prisms with positive rational number dimensions.

Where have we been?

Students begin this topic by building on their previous knowledge of triangles to discover the Triangle Sum theorem. In previous courses, students have learned about area of squares and rectangles. Now, they will decompose rectangles to derive area formulas for parallelograms, triangles, and trapezoids. In Grade 5, students learned how to calculate the volume of a right rectangular prism by filling it with cubes and eventually by using the formulas $V = \ell wh$ and $V = Bh$. Now, students extend that previous understanding to calculating the volume of right rectangular prisms with positive rational number dimensions.

Where are we going?

This topic provides the building blocks for the remaining geometry topics in future courses. In Grades 7 and 8, students will continue to build upon angle relationships as well as area of figures when they decompose composite figures to calculate total area. Students continue building their knowledge of area and volume when they calculate surface area and volume of prisms, pyramids, cylinders, cones, and spheres.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is becoming fluent with fraction operations and gaining experience with the area of two-dimensional shapes and the volume of right rectangular prisms.

QUESTIONS TO ASK

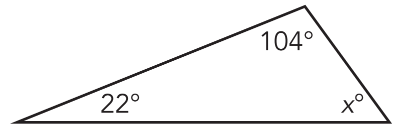
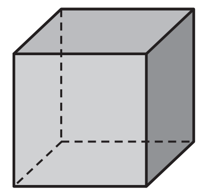
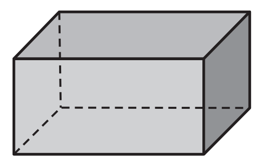
- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? Why or why not?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- Triangle Inequality theorem
- Triangle Sum theorem [teorema de la suma del triángulo]
- parallelogram [paralelogramo]
- variable [variable]
- straightedge
- trapezoid [trapecio]
- geometric solid
- polyhedron [poliedro]
- face
- edge
- vertex [vértice]
- right rectangular prism [prisma rectangular recto]
- cube [cubo]
- volume [volumen]

Refer to the Math Glossary for definitions of New Key Terms.

Where are we now?

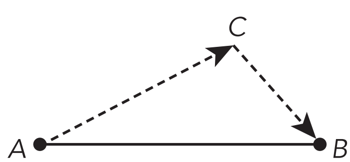
The **Triangle Sum theorem** states that the sum of the measures of the interior angles of a triangle is 180°.	
A **geometric solid** is a bounded three-dimensional geometric figure.	 Cube
A **right rectangular prism** is a **polyhedron** with three pairs of congruent and parallel rectangular **faces**.	

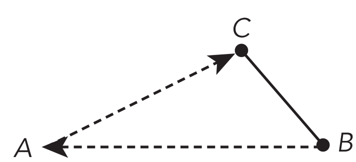
In **Lesson 1: Constructing Triangles Given Sides**, students determine whether three given line segments will create a triangle.

Triangle Inequality Theorem

The **Triangle Inequality theorem** states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$AC + CB > AB \qquad BA + AC > BC$$





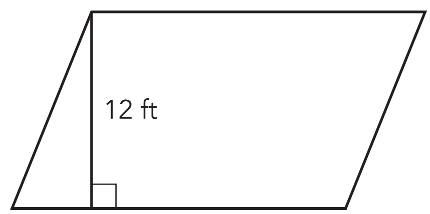
In **Lesson 3: Area of Triangles and Quadrilaterals,** their knowledge of rectangles and area, students also develop the formula, or rule, for determining the area of parallelograms, triangles, and **trapezoids**.

Area of a Parallelogram

A **parallelogram** is a four-sided figure with two pairs of parallel sides with each pair equal in length. In a parallelogram, the height is the distance from the base to the opposite side at a right angle. The area of a parallelogram is equal to $b \cdot h$, where the variable b represents the base and h represents the height. A **variable** is a letter used to represent a number.

For example, in this parallelogram, the base, b, is 20 feet and the height, h, is 12 feet.

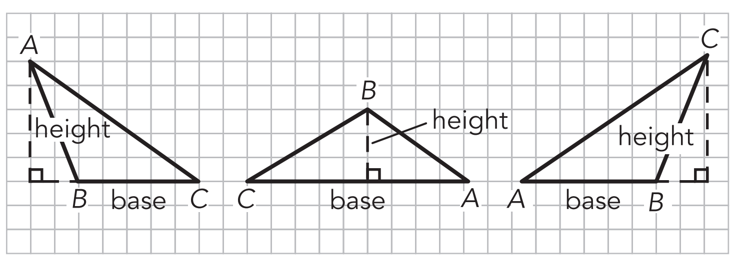
$$\begin{aligned}\text{Area of a parallelogram} &= bh \\ &= (20)(12) \\ &= 240 \text{ square feet}\end{aligned}$$



Area of a Triangle

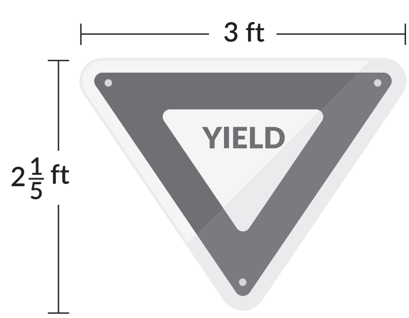
The area of a triangle is equal to $\frac{1}{2}bh$. The base of a triangle can be any of its sides.

Draw a line straight down from the top corner of the triangle to the bottom, or base at a right angle. This is called the height of the triangle.



For example, in this triangle, the base, b, is equal to 3 feet and the height, h, is equal to $2\frac{1}{5}$ feet.

$$\begin{aligned}\text{Area of a triangle} &= \tfrac{1}{2}bh \\ &= \tfrac{1}{2}(3)\left(2\tfrac{1}{5}\right) \\ &= 3\tfrac{3}{10} \text{ square feet}\end{aligned}$$



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MYTH

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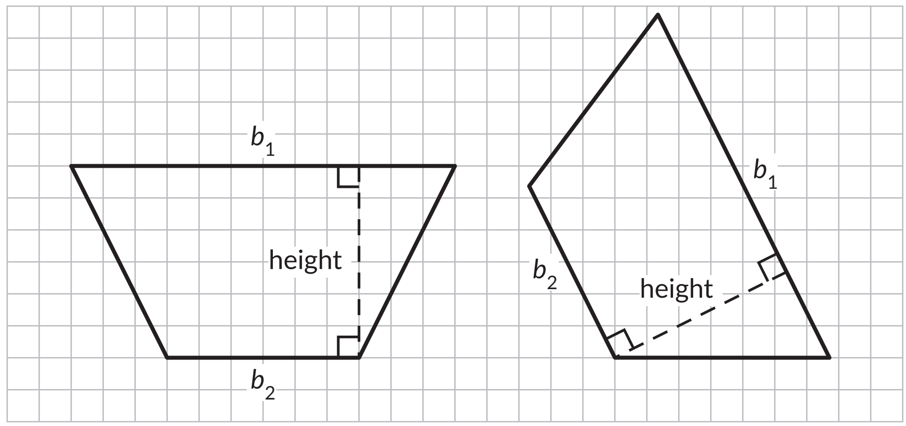
Questions don't always indicate a lack of understanding. Instead, they might allow you to learn even more about a subject that you already understand. Asking questions may also give you an opportunity to ensure that you understand a topic correctly. Finally, questions are extremely important to ask yourself. For example, ***everyone*** should be in the habit of asking themselves, "Does that make sense? How would I explain it to a friend?"

#mathmythbusted

Area of a Trapezoid

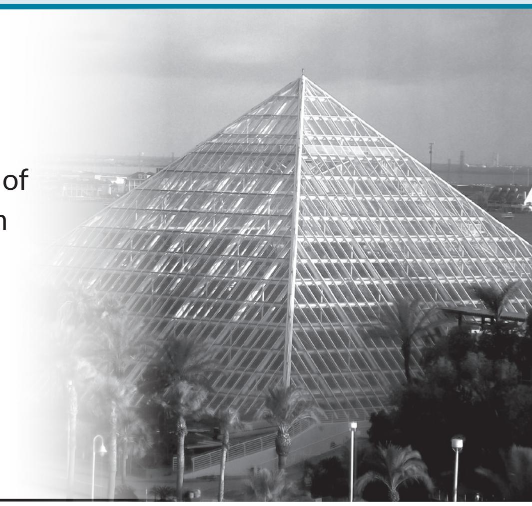
A trapezoid is a quadrilateral with exactly 1 pair of parallel sides. It has two bases that are parallel to each other, often labeled b_1 and b_2. The other two sides of a trapezoid are called the legs of the trapezoid. A height of a trapezoid is the length of the shortest line drawn from one base to the other at a right angle.

The area of a trapezoid is equal to $\frac{1}{2}(b_1 + b_2)h$.



Family Guide

MODULE 1 Composing and Decomposing Grade 6



TOPIC 3 Decimals

In this topic, students review addition and subtraction of decimal numbers and continue operating with decimals, with the eventual goal of fluency. Students also review whole-number and decimal multiplication and learn how to long divide with whole numbers and decimals.

Where have we been?

Students began learning about decimals in Grades 4 and 5. They have experience using concrete models and place-value strategies to operate with decimals to the hundredths place. In Grade 5, students learned how to calculate the volume of a right rectangular prism by filling it with cubes and eventually by using the formulas $V = \ell wh$ and $V = Bh$.

Where are we going?

Students will use decimal operations to solve real-world and mathematical problems throughout the remaining modules of this course. Fractions and decimals are encountered more frequently than whole numbers in daily life, so students should be comfortable and confident solving problems that require operating with these numbers.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Your student is learning to understand and be fluent with decimals and decimal operations. You can further support your student's learning by asking questions about the work they do in class or at home.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

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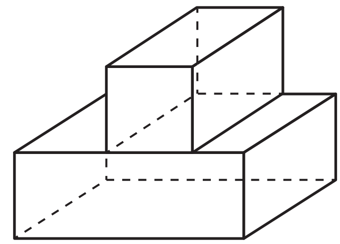
NEW KEY TERMS

- kite
- composite solid
- terminating decimal [decimal terminal]
- repeating decimal [decimal repetitivo]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **composite solid** is made up of more than one geometric solid.



When you write a fraction as a decimal using division, and a digit or a group of digits repeats without end in the quotient, the resulting decimal is a **repeating decimal**.

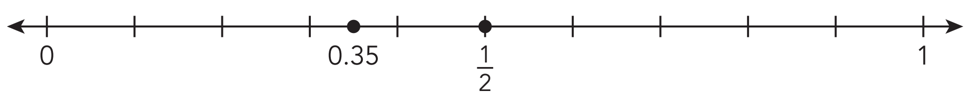
$\frac{1}{9} = 0.111...$ $\quad \frac{7}{12} = 0.58333...$

$\frac{22}{7} = 3.142857142857...$

In **Lesson 1: Plotting, Comparing, and Ordering Rational Numbers**, students plot decimals on a number line, compare, and order decimal values.

Plotting, Comparing, and Ordering Rational Numbers

Compare $\frac{1}{2}$ and 0.35. Which value is greater? First, convert $\frac{1}{2}$ to a decimal. $\frac{1}{2}$ is equivalent to $\frac{5}{10}$, or 0.5. Plot each value on a number line.

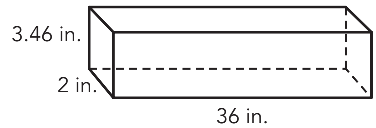


Because $\frac{1}{2}$ is to the right of 0.35 on the number line, $\frac{1}{2}$ is greater than 0.35, or $\frac{1}{2} > 0.35$.

In **Lesson 2: Multiplying Decimals**, students review decimal multiplication.

Multiplying Decimals

A poster is rolled up and mailed in a cardboard box in the shape of a rectangular prism. Determine the volume of the box.



The formula for determining the volume of a rectangular prism is *volume* $=$ *length* $\cdot$ *width* $\cdot$ *height*, or $V = \ell \cdot w \cdot h$. Multiply the three values.
$36 \cdot 2 \cdot 3.46 = 249.12$

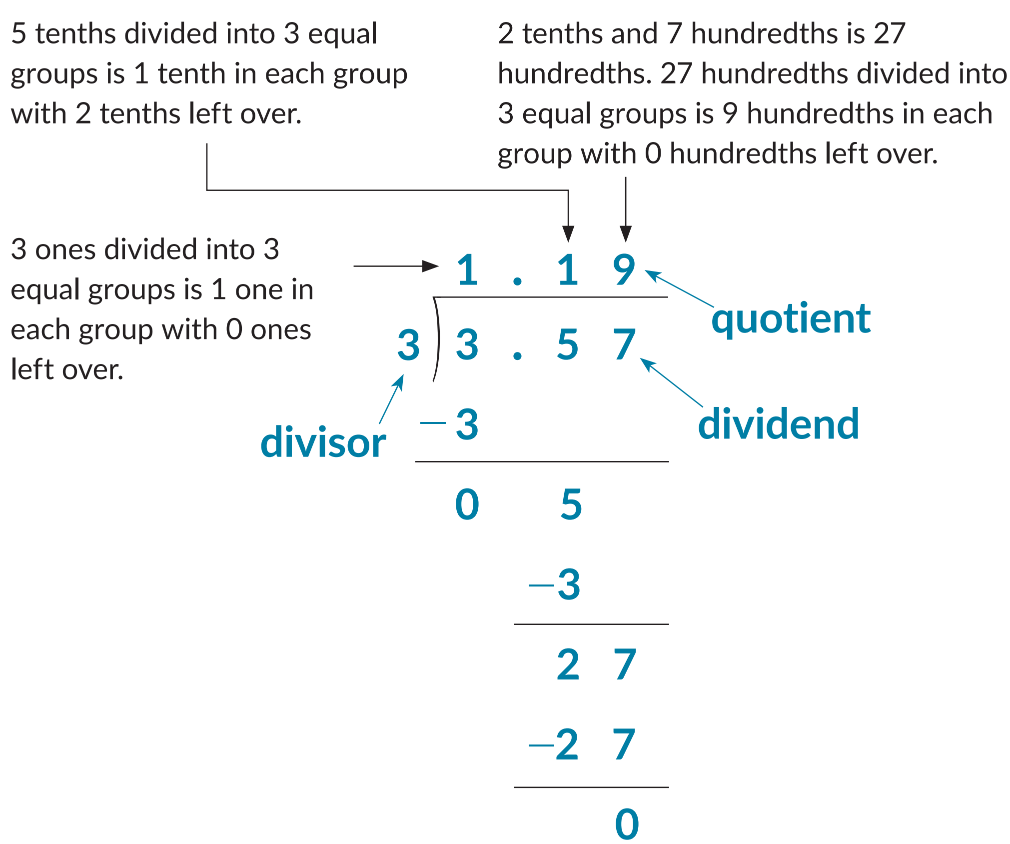
The volume of the box is 249.12 cubic inches.

In **Lesson 3: Dividing Decimals**, students use the standard algorithm for long division with decimals.

Using a Standard Algorithm to Divide Decimals

The long division algorithm uses organized estimation and place value to determine a quotient, or the number of times the divisor is contained in the dividend.

Let's use the standard algorithm to solve $3.57 \div 3$. The dividend is 3.57 and the divisor is 3.



The quotient is 1.19; therefore, $3.57 \div 3 = 1.19$.

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MYTH

Some students are "right-brain" learners while other students are "left-brain" learners.

As you probably know, the brain is divided into two hemispheres: the right and the left. Some categorize people by their preferred or dominant mode of thinking. "Right-brain" thinkers are considered to be more intuitive, creative, and imaginative. "Left-brain" thinkers are said to be more logical, verbal, and mathematical.

The brain can also be broken down into lobes. The *occipital lobe* can be found in the back of the brain, and it is responsible for processing visual information. The *temporal lobes*, which sit above your ears, process language and sensory information. The band across the top of your head is the *parietal lobe*, and it controls movement. Finally, the *frontal lobe* is where planning and learning occur. Another way to think about the brain is from the back to the front, where information goes from highly concrete to abstract.

Why don't we claim that some people are "back of the brain" thinkers, who are highly concrete; whereas, others are "frontal" thinkers, who are more abstract? The reason is that the brain is a highly interconnected organ. Each lobe hands off information to be processed by other lobes, and they are constantly talking to each other. All of us are *whole-brain thinkers!*

#mathmythbusted

MODULE
2

Relating Quantities

Family Guide

MODULE 2 Relating Quantities

Grade 6

TOPIC 1 Ratios and Rates

Students begin this topic by learning about ratios as multiplicative comparisons, contrasting them with additive comparisons. “More than” and “less than” are examples of additive comparisons, whereas “twice as many” and “one half as many” are examples of multiplicative comparisons. Students learn about quantitative relationships represented by ratios and the different ways to represent ratios. They are introduced to percent as a special ratio, an amount per 100. Students use their initial understandings of ratio to model and determine equivalent ratios and rates. To generate and display equivalent ratios and rates in real-world and mathematical problems, they use strip diagrams, double number lines, scaling up and down, tables, and graphs.

Where have we been?

Students enter Grade 6 with experience contrasting additive and multiplicative patterns and relationships. In prior grades, they wrote number sentences to represent multiplicative and additive scenarios. Students’ knowledge of equivalent fractions from previous courses provides the foundation for their developing understanding of equivalent ratios and rates.

Where are we going?

This topic provides the basis for future learning of proportional relationships and slope. Students also graph equivalent ratios and rates on the coordinate plane, a prerequisite for the more in-depth study of proportional relationships in Grade 7 and direct variation in Grade 8.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your students’ learning by asking them to take a step back and think about a different strategy when they are stuck.

QUESTIONS TO ASK

- What strategy are you using?
- What is another way to solve the problem?
- Can you draw a model?
- Can you come back to this problem after doing some other problems?

NEW KEY TERMS

- additive reasoning
- multiplicative reasoning
- ratio
- percent [porcentaje]
- equivalent ratios
- strip diagram
- rate
- proportion [proporción]
- scaling up
- scale factor [factor de escala]
- scaling down
- double number line
- linear relationship [relación lineal]

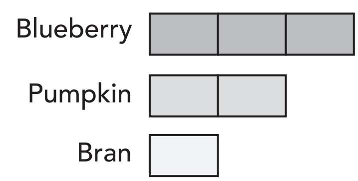
Refer to the Math Glossary for definitions of New Key Terms.

Where are we now?

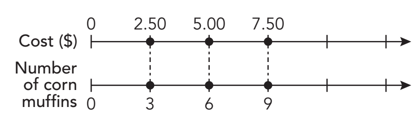
A **ratio** is a multiplicative comparison between two quantities that may contain the same or different units.

The ratio of stars to circles is $\frac{4}{2}$, or 4 : 2, or 4 to 2. You can rewrite this ratio as the **equivalent ratio** 2 : 1. The ratio of circles to stars is $\frac{2}{4}$, or 2 : 4, or 2 to 4. You can rewrite this ratio as the equivalent ratio 1 : 2.

A **strip diagram** represents the ratio of each type of muffin to one another.
A bakery sells packs of muffins in the ratio of 3 blueberry muffins : 2 pumpkin muffins : 1 bran muffin. The strip diagram represents the ratio of each type of muffin to one another.



A **double number line** is a model that is made up of two number lines used together to represent the ratio between two quantities.

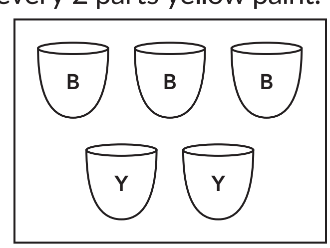


In **Lesson 1: Introduction to Ratio and Ratio Reasoning**, students learn about quantitative relationships represented by ratios and the different ways to represent ratios.

Comparing Quantities

There are different ways to think about relationships and make comparisons. One way is to draw a model.

The school colors at a middle school are a shade of bluish-green and white. The art teacher, Mr. Park, knows that to get the correct color of bluish-green, it takes 3 parts blue paint to every 2 parts yellow paint. From the model, you can make comparisons of the different quantities.



- the number of blue parts to yellow parts; 3 : 2, 3 to 2, or $\frac{3}{2}$
- the number of yellow parts to blue parts; 2 : 3, 2 to 3, or $\frac{2}{3}$
- the number of blue parts to total parts; 3 : 5, 3 to 5, or $\frac{3}{5}$
- the number of yellow parts to total parts; 2 : 5, 2 to 5, or $\frac{2}{5}$

Each comparison is called a *ratio*.

Special Types of Ratios

When a ratio is written using the total number of parts, you are writing a **fraction.** A fraction can be used as a ratio that shows a part-to-whole relationship.

Ratios

part : part   part : whole

$\frac{\text{part}}{\text{part}}$ $\underbrace{\frac{\text{part}}{\text{whole}}}_{\textbf{Fraction}}$

Fractional form simply means writing the relationship in the form $\frac{a}{b}$. Just because a ratio looks like a fraction does not mean it represents a part-to-whole comparison. Only a part-to-whole ratio is a fraction.

A **percent** is a part-to-whole ratio, where the whole is equal to 100. Percent is another name for hundredths. The percent symbol "%" means "out of 100."

Rate

A **rate** is a comparison by division between two quantities that have different units.

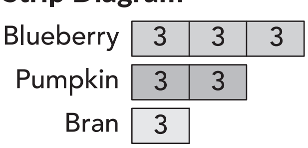
For example, Luna can answer 4 problems correctly in five minutes.

This rate is $\frac{\text{4 problems correct}}{\text{5 minutes}}$.

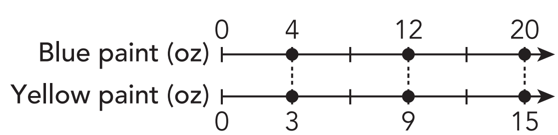
In **Lesson 3: Determining Equivalent Ratios and Rates**, **Lesson 4: Using Tables to Represent Equivalent Ratios and Rates**, and **Lesson 5: Graphs of Ratios and Rates**, students use different models of equivalent ratios. They reason why these models work.

Equivalent Ratios and Rates

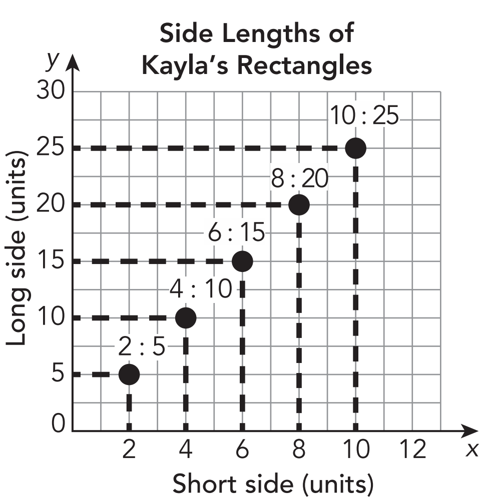
Strip Diagram



Double Number Line



Graph



Ratio Table

Weight on Earth (lb)	60	30	90
Weight on the moon (lb)	10	5	15

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MYTH

There is one right way to do math problems.

Employing multiple strategies to arrive at a single, correct solution is important in life. Suppose you are driving in a crowded downtown area. If one road is backed up, then you can always take a different route. If you know only one route, then you're out of luck.

Learning mathematics is no different. There may only be one right answer, but there are often multiple strategies to arrive at that solution. Everyone should get in the habit of saying: *Well, that's one way to do it. Is there another way? What are the pros and cons?* That way, you avoid falling into the trap of thinking there is only **one** right way, because that strategy might not always work, or there might be a more efficient strategy.

Teaching students multiple strategies is important. This helps students understand the benefits of the more efficient method. In addition, everyone has different experiences and preferences. What works for you might not work for someone else.

#mathmythbusted

Lesson 3: Determining Equivalent Ratios, introduces students to proportions and how they can use their knowledge of equivalent ratios to scale up or down to solve problems.

Proportions

When two **ratios** or **rates** are equal to each other, they can be written as a proportion. A **proportion** is an equation that states that two ratios are equal. In a proportion, the quantities composing each part of the ratio have the same multiplicative relationship between them.

To change a ratio or rate to an equivalent ratio or rate you can scale up or scale down.

Scaling up means to multiply both parts of the ratio or rate by the same factor greater than 1.

$$\frac{\text{2 blueberry muffins}}{\text{5 total muffins}} = \frac{\text{20 blueberry muffins}}{\text{? total muffins}}$$

Scale up the ratio to determine the unknown number.

What factor did you use to scale up the ratio?

[*50 total muffins*; *You can use a factor of 10*.]

Scaling down means you divide both parts of the ratio or rate by the same factor greater than 1, or you multiply both parts of the ratio or rate by the same factor less than 1.

$$\frac{\text{20 hours of work}}{\$240} = \frac{\text{1 hour of work}}{?}$$

Scale down the rate to determine the unknown number. What factor did you use to scale down the rate?

[*$12; You can use a factor of 20*.]

Family Guide

MODULE 2 Relating Quantities — Grade 6

TOPIC 2 Percents

In *Percents*, students transition from thinking about ratio relationships in general to focusing on a special ratio relationship: percent. Students learn that a percent can be defined multiple ways: as a ratio, as a decimal to the hundredths place, and as a part-to-whole relationship in which the whole is 100. Students use their knowledge of fractions and decimals and their intuitive understanding of percents to write and compare rational numbers written in these three different forms. They complete number lines of common fractions, decimals, and percent equivalences, connecting to prior work with benchmark fractions and decimals. Throughout this topic, students continue to develop their fluency with whole numbers, fractions, decimals, area, and volume in the context of solving mathematical and real-world problems.

Where have we been?

Students have used the relationship between decimals and fractions to write decimals as fractions, and they have used benchmark fractions and decimals to understand ordering of numbers. This topic provides students with similar experiences using this new representation: *percents*. Because percent is a special ratio, students continue to use the strategies and reasoning developed in the prior topic to solve percent problems.

Where are we going?

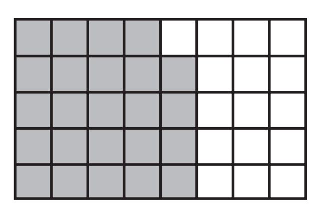
Percents are very useful, not only in mathematics but in everyday life and work. In Grade 7, students will use the foundation they establish here to solve more advanced percent problems, including problems involving discounts, tax, interest, percent increase or decrease, and tips.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

A common error that students make when working with part-to-whole ratios (like percents and fractions) is to forget about the whole. Look for ways to remind your student about this common mistake.

For example, this model shows 24 shaded squares. Students might say that 24% of the model is shaded.



But the whole is not 100, it's 40. So, $\frac{24}{40}$, or 60%, is shaded. Also, more than half is shaded, so it has to be more than 50%.

NEW KEY TERM

- benchmark percent

Refer to the Math Glossary for definitions of the New Key Term.

Where are we now?

A **benchmark percent** is a percent that is commonly used, such as 1%, 5%, 10%, 25%, $33\frac{1}{3}$%, 50%, and 100%.

In **Lesson 1: Percent, Fraction, and Decimal Equivalence**, students learn about the relationships among percents, fractions, and decimals.

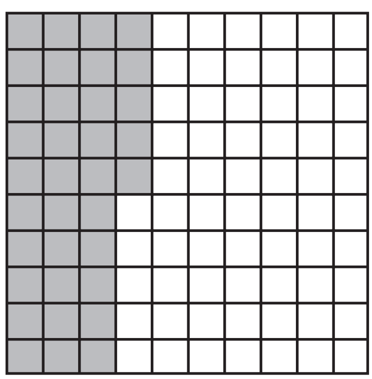
Percents, Fractions, and Decimals

In this topic, students focus on a special ratio relationship: percent. Students learn that a percent can be defined multiple ways: as a ratio, as a decimal to the hundredths place, and as a part-to-whole relationship in which the whole is 100.

Your student will learn that a **percent** is a part-to-whole ratio where the whole is equal to 100.

35% means 35 out of 100.
35% as a fraction is $\frac{35}{100}$.
35% as a decimal is 0.35.
35% as a ratio is 35 to 100, or 35 : 100.

You can shade 35 of the 100 squares on the hundredths grid to represent 35%.



When the denominator is a factor of 100, scale up the fraction to write it as a percent.

$$\underset{\cdot 20}{\overset{\cdot 20}{\frac{4}{5} = \frac{80}{100}}}$$

$$\frac{80}{100} = 80\%$$

When the denominator is not a factor of 100, you can divide the numerator by the denominator to write the fraction as a decimal, which you can then write as a percent.

$$\frac{5}{8} = 5 \div 8$$

$$\begin{array}{r}
0.625 \\
8\overline{)5.000} \\
-\ 48 \\
\hline
20 \\
-\ 16 \\
\hline
40 \\
-\ 40 \\
\hline
0
\end{array}$$

$$0.625 = 62.5\%$$

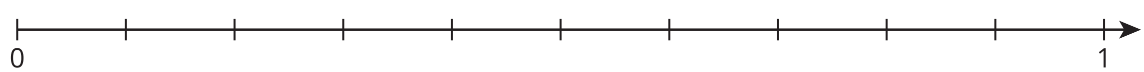
Also in this topic, your student will complete number lines of common fractions, decimals, and percent equivalences.

Label each mark on the number line with a fraction, decimal, and percent. Make sure your fractions are in lowest terms.

Remember to:

- work carefully and check your work.
- calculate accurately and communicate precisely to others.

Example:



Fraction	0	$\frac{1}{10}$	______	______	$\frac{2}{5}$	______	______	$\frac{7}{10}$	______	______	1
Decimal	0.0	______	0.2	______	______	0.5	______	______	______	0.9	1.0
Percent	0%	______	______	30%	______	______	60%	______	80%	______	100%

In **Lesson 2: Using Estimation and Benchmark Percents**, students are introduced to benchmark percents to help students mentally estimate the value of a percent.

Benchmark Percents

A **benchmark percent** is a percent that is commonly used, such as 1%, 5%, 10%, 25%, $33\frac{1}{3}$%, 50%, and 100%. With fractions and decimals, benchmarks can be used to make estimations. Your students can use benchmarks to calculate any whole percent of a number.

You can determine any whole percent of a number by using 10%, 5%, and 1%.

For example, what is 26% of 300?

$26\% = 10\% + 10\% + 5\% + 1\%$

10% of 300 is $300 \cdot \frac{1}{10}$, or 30.

5% of 300 is $30 \cdot \frac{1}{2}$, or 15.

1% of 300 is $15 \cdot \frac{1}{5}$, or 3.

$10\% + 10\% + 5\% + 1\%$

$30 + 30 + 15 + 3 = 78$

26% of 300 is 78.

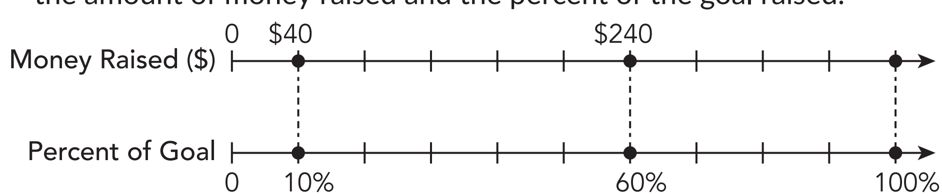
In **Lesson 3: Determining the Part and the Whole in Percent Problems**, students construct a double number line to solve percent problems.

Using Double Number Lines to Solve Percent Problems

Percent problems often have a part, a percent, and a whole. When you know the part and the percent, you can use different strategies to determine the whole.

One strategy is a double number line.

For example, Nahimana's homeroom raised $240 for charity, which is 60% of its goal. Nahimana uses a double number line to record the amount of money raised and the percent of the goal raised.



To determine the value that corresponds to 10%, Nahimana divided the amount raised so far by 6: $\$240 \div 6 = \$40$.

Since $10\% \cdot 10 = 100\%$, she can multiply $40 by 10 to determine the homeroom's goal: $\$40 \cdot 10 = \$400$.

MYTH

Students only use 10% of their brains.

Hollywood is in love with the idea that humans only use a small portion of their brains. This notion formed the basis of some science fiction movies that ask the audience: *Imagine what you could accomplish if you could use 100% of your brain!*

Well, this isn't Hollywood. The good news is that you ***do*** use 100% of your brain. As you look around the room, your *visual cortex* is busy assembling images, your *motor cortex* is busy moving your neck, and all of the *associative areas* recognize the objects that you see. Meanwhile, the *corpus callosum*, which is a thick band of neurons that connect the two hemispheres, ensures that all of this information is kept coordinated. Moreover, the brain does this automatically, which frees up space to ponder deep, abstract concepts . . . like mathematics!

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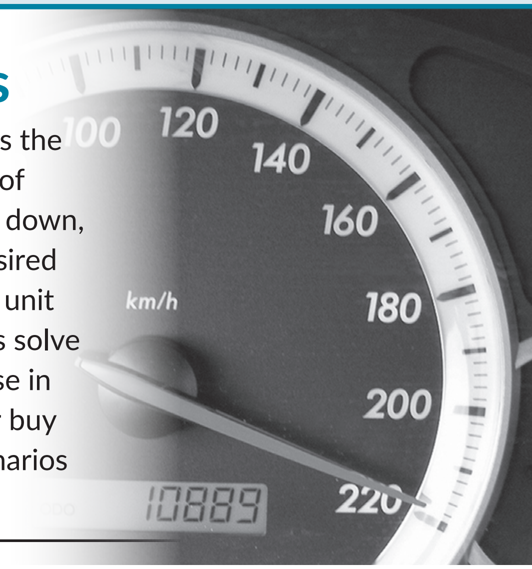
Family Guide

MODULE 2 Relating Quantities Grade 6

TOPIC 3 Unit Rates and Conversions



Students learn that converting within systems of measurement involves the use of conversion rates, another special type of ratio. To convert units of measurement, they use double number lines, ratio tables, scaling up or down, and unit analysis. Students use multiple conversions to convert to a desired unit. They use models to illustrate the meaning of a unit rate and write unit rates that compare the same quantities in two different ways. Students solve a variety of unit rate problems, determining which unit rates make sense in the context of a problem. They evaluate prices to determine the better buy and solve problems involving constant speed. Finally, they analyze scenarios and clearly identify the unit rates from tables and graphs.

Where have we been?

Students enter Grade 6 with experience converting among different-sized standard units within a given system of measurement. Students use strategies from the previous topics, tables and double number lines, to complete conversions, moving from multiplicative reasoning to ratio strategies to unit analysis. As students continue in the topic, they use all of the strategies developed in the previous topic to solve unit rate problems.

Where are we going?

Unit Rates and Conversions provides the foundation for important ideas in algebra and science: slope and dimensional analysis. In Grade 7, students will use their understanding of unit rate to convert between systems of measurement and represent proportional relationships between quantities. They will use unit rate to write equations and graph proportional relationships, developing an informal understanding of slope.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your students' learning by asking questions about the work they do in class or at home. Your student is learning to work with unit rates and measurement conversions.

SOME THINGS TO LOOK FOR

Look for real-life examples of conversions, like inches to feet, days to years. Any time you ask about a measurement in different units, you'll need to do a conversion with ratios and rates. When your student converts a measurement to different units, ask them to explain whether their answer makes sense.

NEW KEY TERMS

- convert [convertir]
- unit rate

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

To **convert** a measurement means to change it to an equivalent measurement in different units.	To convert 36 inches to feet, you can multiply: $36\ \cancel{\text{in.}}\left(\frac{1\text{ ft}}{12\ \cancel{\text{in.}}}\right) = \frac{36\text{ ft}}{12}$ $= 3\text{ ft}$
A **unit rate** is a comparison of two different measurements in which the numerator or denominator has a value of one unit.	The speed 60 miles in 2 hours can be written as a unit rate: $\frac{60\text{ mi}}{2\text{ h}} = \frac{30\text{ mi}}{1\text{ h}}$. The unit rate is 30 miles per hour.

In **Lesson 1: Using Ratio Reasoning to Convert Units**, students deepen their understanding of converting units of measurement through the use of ratio reasoning and strategies for determining equivalent ratios.

Unit Conversions

Students learn that converting within systems of measurement involves the use of conversion rates, another special type of ratio. To convert units of measurement, they use double number lines, ratio tables, scaling up or down, and unit analysis.

The table below lists common conversions written as ratios. They are also listed symbolically using the equal sign.

Ratio Language	Symbolically
For every 12 inches, there is exactly 1 foot.	12 in. = 1 ft
For every yard, there are 36 inches.	1 yd = 36 in.
For every mile, there are 1760 yards.	1 mi = 1760 yd
For every meter, there are 1000 millimeters.	1 m = 1000 mm
For every 1 kilometer, there are exactly 1000 meters.	1 km = 1000 m

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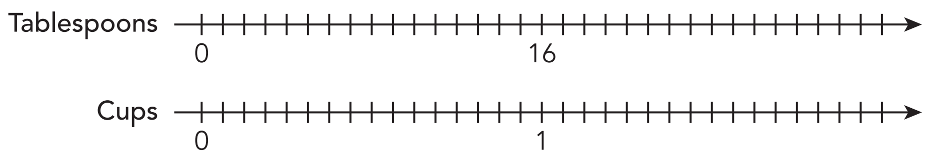
Using Double Number Lines to Convert Units

When your student learned about ratios and percents in the previous topics, they learned to use double number lines to determine equivalent ratios. Double number lines can also be used to convert from one unit to another.

For example, you are baking cookies at your friend's house. After searching the cupboards and drawers, you cannot find the measuring cups, but you can find the tablespoon.

Shown on the double number line is the conversion rate for tablespoons and cups. There are 16 tablespoons in 1 cup.



Use the double number line to determine how many tablespoons are equal to 2 cups and $\frac{1}{2}$ cup.

Use the double number line to determine the number of cups equal to 24 tablespoons.

Using a Ratio Table to Convert Units

Using a ratio table is another strategy for converting units. For example, this table represents the ratio of pounds to ounces.

Pounds	1	2	$\frac{1}{4}$	$1\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$2\frac{1}{2}$
Ounces	16	32	4	20	8	6	40

You can add values in different columns to determine new equivalent rates.

32 oz + 8 oz = 40 oz, so 2 lb + $\frac{1}{2}$ lb = $2\frac{1}{2}$ lb.

BY NC

MYTH

Just watch a video, and you will understand it.

Has this ever happened to you? Someone explains something, and it all makes sense at the time. You feel like you get it. But then, a day later when you try to do it on your own, you suddenly feel like something's missing? If that feeling is familiar, don't worry. It happens to us all. It's called the *illusion of explanatory depth*, and it frequently happens after watching a video.

How do you break this illusion? The first step is to try to make the video interactive. Don't treat it like a TV show. Instead, pause the video and try to explain it to yourself or to a friend. Alternatively, attempt the steps in the video on your own and rewatch it if you hit a wall. Remember, it's easy to confuse *familiarity* with *understanding*.

#mathmythbusted

Scaling Up or Down to Convert Units

Scaling up or down is a similar strategy for determining equivalent ratios that can more easily be used to convert from one unit of measurement to another.

For example, you can use scaling up to determine how many ounces are in 2.5 pounds. Because you want to determine the number of ounces for a specific number of pounds, use the conversion rate 1 lb = 16 oz or $\frac{1\text{ lb}}{16\text{ oz}}$.

Scaling up

$$\frac{1\text{ lb}}{16\text{ oz}} = \frac{2.5\text{ lb}}{?\text{ oz}} \longrightarrow \frac{1\text{ lb}}{16\text{ oz}} \overset{\cdot\, 2.5}{\underset{\cdot\, 2.5}{=}} \frac{2.5\text{ lb}}{40\text{ oz}}$$

In **Lesson 2: Introduction to Unit Rates**, students are introduced to unit rates.

Unit Rate

Students model the meaning of a unit rate. They solve different unit rate problems, determining which unit rates make sense in a situation.

For example, unit rates are helpful when solving problems about constant speeds. Suppose Jasmine can ride 50 miles in 4 hours. At this rate, how far will she ride in 7 hours?

Scale down to determine the unit rate.

$$\frac{50\text{ miles}}{4\text{ hours}} = \frac{12.5\text{ miles}}{1\text{ hour}}$$

Then scale up to determine the equivalent rate needed to solve the problem.

$$\frac{12.5\text{ miles}}{1\text{ hour}} = \frac{87.5\text{ miles}}{7\text{ hours}}$$

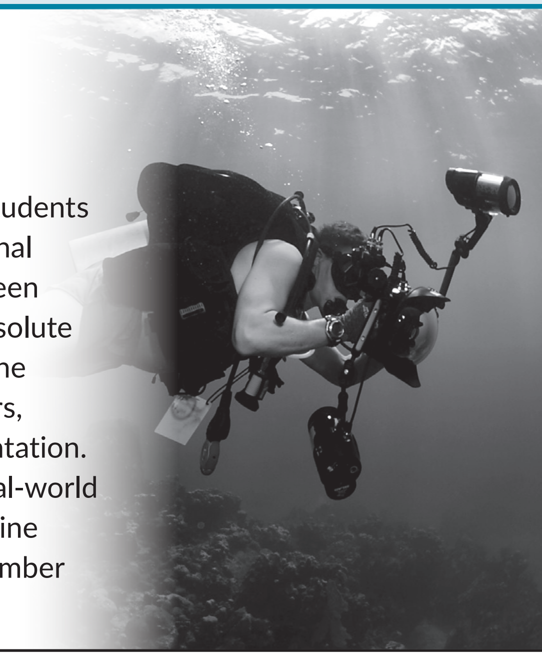
MODULE
3

Moving Beyond Positive Quantities

Family Guide

MODULE 3 Moving Beyond Positive Quantities Grade 6

TOPIC 1 Signed Numbers and The Four Quadrants



In this topic, students are formally introduced to negative numbers. Students begin by reflecting the positive numbers across zero to build the rational number line. They develop an understanding of the relationship between opposites and distance on a number line, leading to the concept of absolute value. Now that students understand negative numbers, they revisit the number system to explore relationships among sets of natural numbers, whole numbers, integers, and rational numbers using a visual representation. They order sets of rational numbers arising from mathematical and real-world contexts. Then, students use their knowledge of the rational number line to build their own four-quadrant coordinate plane and plot rational number coordinates including integers, decimals, and percents.

Where have we been?

Prior to Grade 6, students positioned positive numbers, including fractions and decimals, on number lines and operated with these numbers using number lines as references. They also represented real-world and mathematical problems in the first quadrant of a coordinate plane and interpreted the coordinate values of points. This topic will build off that foundation as students learn about the full rational number line and the four-quadrant coordinate plane with the formal introduction of negative numbers.

Where are we going?

The foundation provided in this topic will enable students to develop strategies for operating with signed numbers in the next topic of this course. This topic provides students with an introduction to the entire real number coordinate plane, which they will continue to use throughout this course and future courses to represent relationships and interpret meanings of points, lines, and other graph elements.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by resisting the urge, as long as possible, to get to the answer in a problem that your student is working on. Students are encountering negative numbers formally for the first time in this topic. They will need time and space to struggle with all the implications of working with this expanded number system. Practice asking good questions when your student is stuck.

QUESTIONS TO ASK

- Let's think about this. What are all the things you know?
- What do you need to find out?
- How can you model this problem?

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NEW KEY TERMS

- negative numbers [números negativos]
- infinity [infinito]
- absolute value [valor absoluto]
- integers [enteros]
- ellipsis
- rational numbers [números racionales]
- density property [propiedad de densidad]
- quadrants [cuadrantes]

Refer to the Math Glossary for definitions of the New Key Terms.

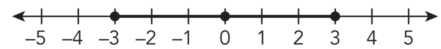
NEW SYMBOL

Symbol	Description
∞	Infinity symbol

Where are we now?

The **absolute value**, or magnitude, of a number is its distance from zero on a number line.

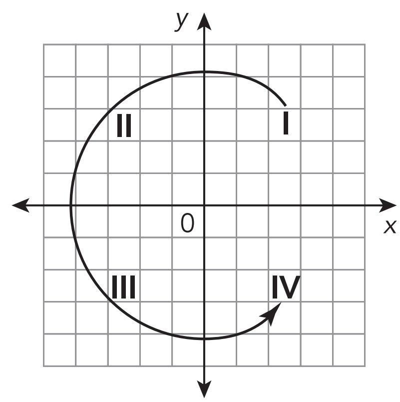
The absolute value of −3 is the same as the absolute value of 3 because they are both a distance of 3 from zero on a number line.



$|-3| = |3|$

The *x*- and *y*-axes divide the coordinate plane into four regions called **quadrants**.

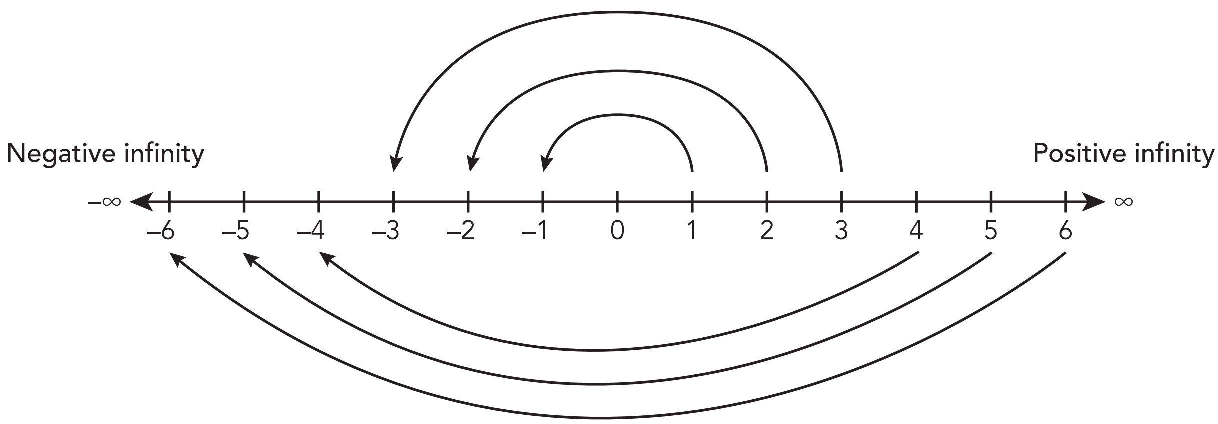
These quadrants are numbered with Roman numerals from one (I) to four (IV), starting in the upper right-hand quadrant and moving counterclockwise.



In **Lesson 1: Introduction to Negative Numbers**, students extend their understanding of numbers to include negatives by building on prior knowledge of ordering positive rational numbers and plotting them on a number line.

Negative Numbers

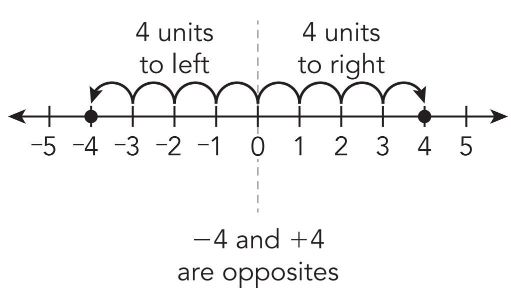
In this topic, students are introduced to **negative numbers**. Students begin by reflecting the positive numbers across zero to build the rational number line. The values to the left of zero on the number line are called *negative numbers*.



In **Lesson 2: Absolute Value**, students formalize the idea that opposites are the same distance from zero and call this distance the absolute value of a number.

Opposites and Absolute Value

Students learn that opposite numbers are reflections of each other across 0 on the number line. The opposite of a positive number is a corresponding negative number. The opposite of a negative number is a corresponding positive number. The **absolute value**, or magnitude, of a number is its distance from zero on a number line. A number and its opposite have the same absolute value.



The absolute value of 4 is 4.

The absolute value of −4 is 4.

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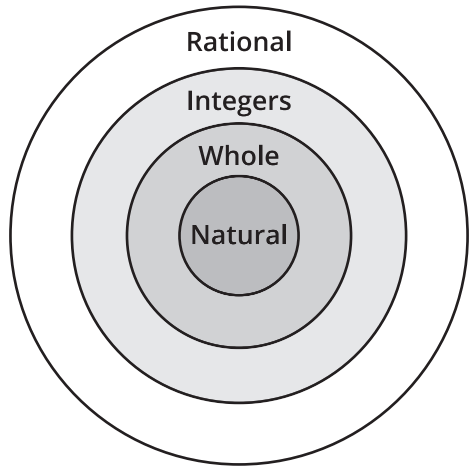
In **Lesson 3: Rational Number System**, students formally classify numbers as rational numbers and understand that all numbers they have studied so far are subsets of the rational numbers.

Classifying Numbers

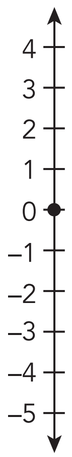
Your student will study the number system to learn more about the relationships among sets of numbers.



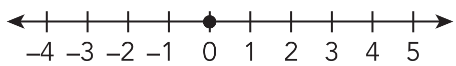
	Natural Numbers	Whole Numbers	Integers	Rational Numbers
Examples	{1, 2, 3, ...}	{0, 1, 2, 3, ...}	{..., −3, −2, −1, 0, 1, 2, 3, ...}	$\left\{-4, 3\%, 2\frac{3}{5}, \frac{11}{3}, \frac{1}{2}, 0.35, 5\right\}$
Description	Counting numbers	Natural numbers and 0	Whole numbers with their opposites	Numbers that you can write as $\frac{a}{b}$, where a and b are integers and $b \neq 0$

The Coordinate Plane

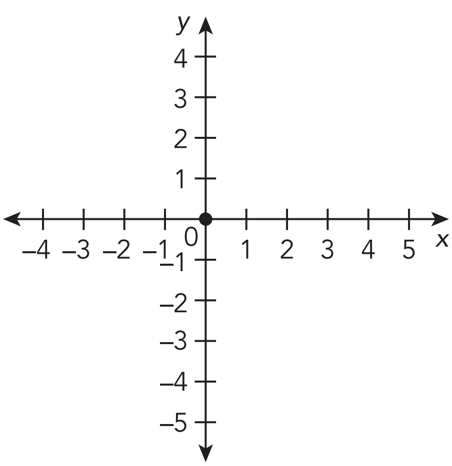
Students use reflections of the first quadrant and their knowledge of the rational number line to build their own four-quadrant coordinate plane. A coordinate plane is made up of two number lines, a horizontal number line and a vertical number line, that meet at the zeros. Zero on a number line or coordinate plane is called the *origin*. Students will plot rational number coordinates that include integers, fractions, and decimals on the coordinate plane.



Vertical Number Line



Horizontal Number Line



The Coordinate Plane

MYTH

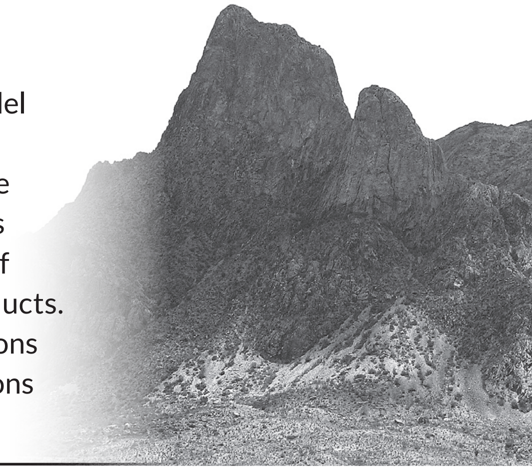
Cramming for a test is just as good as spaced practice for long-term retention.

Everyone has been there. You have a big test tomorrow, but you've been so busy that you haven't had time to study. So you had to learn it all in one night. You may have received a decent grade on the test. However, did you remember the material a week, a month, or a year later?

The honest answer is, "probably not." That's because long-term memory is designed to retain useful information. How does your brain know if a memory is "useful" or not? One way is the frequency in which you encounter a piece of information. If you see something only once (like during cramming), then your brain doesn't deem those memories as important. However, if you sporadically come across the same information over time, your brain is more likely to recognize it as important. To optimize retention, encourage your student to periodically study the same information over expanding intervals of time.

#mathmythbusted

TOPIC 2 Operating with Integers



In this topic, students use number lines and two-color counters to model addition, subtraction, and multiplication of integers to gain conceptual understanding before developing the abstract rules for determining the sum, difference, and product of signed numbers. Students use patterns and multiplication fact families to develop the rules for the quotients of signed numbers, namely that the same rules apply to quotients as products. Students are expected to make connections between the representations used. Students then apply their understanding of operations to questions with three or more integers.

Where have we been?

In the previous topic, students learned how to represent positive and negative rational numbers on a number line. In previous courses, students studied fact families for adding and subtracting numbers, as well as multiplying and dividing numbers. They will use all of these skills in this topic as they learn to operate with integers on the number line and generalize rules about the signs of sums, differences, products, and quotients of integers.

Where are we going?

Students will develop a strong conceptual foundation for operating with integers so they can apply those skills to the full set of rational numbers beyond Grade 6. This foundation serves as a basis for manipulating and representing increasingly complex numeric and algebraic expressions and equations in future courses.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to reason using signed numbers.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

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NEW KEY TERMS

- additive inverse property [propiedad del inverso aditivo]
- additive inverses [inversos aditivos]
- zero pair [par cero]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

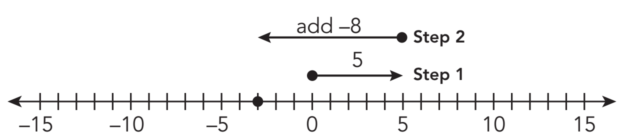
Two numbers with the sum of zero are called **additive inverses**.	$-19 + 19 = 0$ $a + (-a) = 0$
A positive counter and a negative counter together make a **zero pair**, since the total value of the pair is zero.	$\oplus + \ominus = 0$

In **Lesson 2: Adding Integers, Part I; Lesson 4: Subtracting Integers;** and **Lesson 5: Multiplying and Dividing Integers**, students use number lines to model addition, subtraction, and multiplication of integers.

Number Lines

Using Number Lines to Add

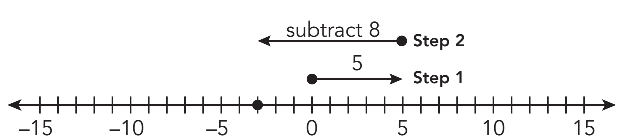
The number line shows how to determine $5 + (-8)$.



The absolute value of 5 is 5, and the absolute value of -8 is 8. The -8 has a greater absolute value, or distance from zero. Therefore, the sign of the sum is negative.

Using Number Lines to Subtract

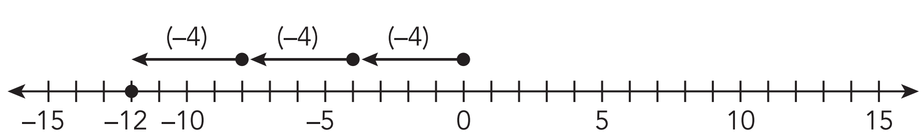
Consider the expression $5 - 8$.



First, move to 5. Then move in the opposite direction of subtracting 8. Notice that $5 + (-8)$ and $5 - 8$ have the same solution of -3.

Using Number Lines to Multiply

Consider the expression $3(-4)$. You can think of $3(-4)$ as 3 groups of (-4).

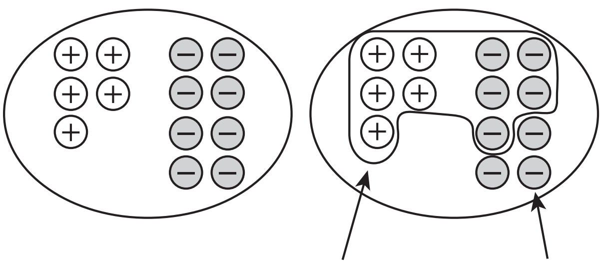


In **Lesson 3: Adding Integers, Part II; Lesson 4: Subtracting Integers;** and **Lesson 5: Multiplying and Dividing Integers**, students use two-color counters to model addition, subtraction, and multiplication of integers before developing rules for determining the sum, difference, and product of positive and negative numbers.

Two-Color Counters

Using Two-Color Counters to Add

Consider the expression 5 + (−8).



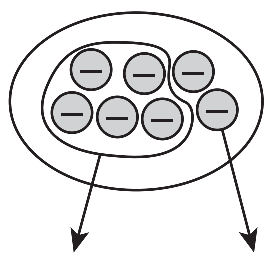
There are five (+) (−) pairs.

The value of those pairs is 0.

There are 3 (−), or negative counters, remaining.

There are 3 negative counters remaining. The sum of 5 + (−8) is −3.

Using Two-Color Counters to Subtract



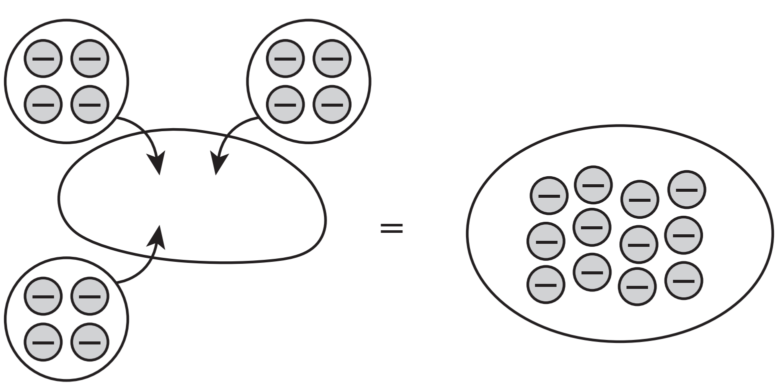
Consider the expression −7 − (−5).
First, start with seven negative counters.
Then, take away five negative counters.
Two negative counters remain.

$$-7 - (-5) = -2$$

You can change any subtraction problem to an addition problem without changing the answer. Subtracting two integers is the same as adding the opposite of the subtrahend, which is the number you are subtracting.

Using Two-Color Counters to Multiply

Consider the expression 3(−4). As repeated addition, it is represented as (−4) + (−4) + (−4).



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MYTH

Asking questions means you don't understand.

The word *smart* is tricky because it means different things to different people. For example, would you say a baby is "smart"? On the one hand, a baby is helpless and doesn't know anything. But on the other hand, a baby is insanely smart because they are constantly learning new things every day!

This example is meant to demonstrate that *smart* can have two meanings. It can mean *the knowledge that you have*, or it can mean *the capacity to learn from experience*. When someone says they are "not smart," are they saying they do not have much knowledge, or are they saying that they lack the capacity to learn? If it's the first definition, then none of us are smart until we acquire information. If it's the second definition, then we know this is completely untrue because everyone has the capacity to grow as a result of new experiences.

So, encourage your student to be patient. They have the capacity to learn new facts and skills. It might not be easy, and it will take some time and effort, but the brain is automatically wired to learn. Smart should not refer only to how much knowledge you currently have.

#mathmythbusted

Integer Rules

Adding Integers

When two integers have the **same sign** and are added together, the sign of the sum is the sign of both integers.

$$6 + 3 = 9$$
$$-6 + (-3) = -9$$

When two integers have **opposite signs** and are added together, the absolute values of the integers are subtracted and the sign of the sum is the sign of the integer with the greater absolute value.

$$-6 + 3 = -3$$
$$6 + (-3) = 3$$

Subtracting Integers

You can change any subtraction problem to an addition problem without changing the answer. Subtracting two integers is the same as adding the opposite of the subtrahend, which is the number you are subtracting.

$$-6 \mathbf{-} (\mathbf{-3}) = -6 \mathbf{+ 3} = -3$$

Multiplying and Dividing Integers

To multiply and divide integers, perform the usual multiplication and division algorithms or steps, and then use the correct sign for the product or quotient. An odd number of negative signs in the expression gives a negative product or quotient. An even number of negative signs in the expression gives a positive product or quotient.

$$-6 \cdot 3 = -18 \qquad -6 \cdot (-3) = 18$$
$$3(-6) = -18 \qquad -3(-6) = 18$$
$$-18 \div (-6) = 3 \qquad 18 \div (-6) = -3$$
$$-18 \div 3 = -6 \qquad 18 \div (-3) = -6$$

Putting It All Together

Students apply their understanding of the rules they create to three or more integers and to questions which include multiple operations.

$$(-14)(-2)(3) = 84$$
$$-6 - (-3) + 9 - 2 = 4$$

MODULE
4

Determining Unknown Quantities

Family Guide

MODULE 4 Determining Unknown Quantities — Grade 6

TOPIC 1 Expressions

In this topic, students develop their understanding of variables and algebraic expressions. They also formalize their knowledge of powers and evaluate expressions involving whole number exponents, expanding their application of the order of operations to include exponents. Students compose algebraic expressions from verbal statements, decompose expressions into their component terms, and evaluate algebraic expressions for given values of the variable. They use algebra tiles and properties of numbers and operations to form equivalent expressions. Students also use tables and graphs to determine whether expressions are equivalent and write algebraic expressions to model and solve real-world and mathematical problems.

Where have we been?

Students enter this course with knowledge of factors and properties of numbers. They have used the commutative and associative properties, as well as the order of operations, although formal terminology may not have been used. These properties, along with the distributive property, have been reviewed in previous lessons. In previous courses, students wrote expressions with whole number exponents for powers of 10, and they wrote numeric expressions to record verbal descriptions of calculations.

Where are we going?

This topic provides the foundation for future work with algebraic structures, including algebraic equations and inequalities and their representations. Expressions are the foundation of equations. Expertise in writing expressions enables students to write and solve equations for many real-world and mathematical problems. In future courses, students must be able to evaluate expressions and determine whether expressions are equivalent.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can support your student's learning by resisting the urge, as long as possible, to get to the answer in a problem that your student is working on. Students will learn the algebraic shortcuts that you may know about but only once they have experience in mathematical reasoning. This may seem to take too long at first. But if you practice asking good questions instead of helping your student arrive at the answer, they will learn to rely on their own knowledge, reasoning, patience, and endurance when struggling with math.

QUESTIONS TO ASK

- What strategy are you using?
- What is another way to solve the problem?
- Can you draw a model?
- Can you come back to this problem after doing some other problems?

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NEW KEY TERMS

- perfect square
- perfect cube [cubo perfecto]
- evaluate a numeric expression [evaluar una expresión numérica]
- order of operations [orden de las operaciones]
- variable [variable]
- algebraic expression [expresión algebraica]
- coefficient [coeficiente]
- term [término]
- evaluate an algebraic expression [evaluar una expresión algebraica]
- like terms
- equivalent expressions [expresiones equivalentes]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

To **evaluate a numeric expression** means to rewrite the expression as a single numeric value.	$19 - 4 \cdot 3$ $19 - 12$ 7
A **variable** is a letter or symbol that is used to represent a number.	$3x = 81$ $\frac{4}{p}$ z^2 variables
In an algebraic expression, **like terms** are two or more terms that have the same variable raised to the same power.	like terms $4x + 3p + x + 2 = 5x + 3p + 2$ like terms $24a^2 + 2a - 9a^2 = 15a^2 + 2a$ no like terms $m + m^2 - x + x^3$

In **Lesson 1: Evaluating Numeric Expressions**, students evaluate numeric expressions, which means to simplify the expression to a single numeric value.

Order of Operations

The **order of operations** is the order in which operations are performed when evaluating any numeric expression.

Order of Operations Rules

1. Evaluate expressions inside the parentheses or grouping symbols.
2. Evaluate exponents.
3. Multiply or divide from left to right.
4. Add and subtract from left to right.

Model

6	4
6	4
6	4

Expression

$3 \cdot (6 + 4)$

or

$(6 + 4) \cdot 3$

3 groups of $(6 + 4)$

3 times the quantity of $(6 + 4)$

Evaluate the expression

	$3 \cdot (6 + 4)$
Parentheses first	$3 \cdot (\mathbf{6 + 4})$
	$3 \cdot \mathbf{10}$
Then, multiply.	30

There are times when students will see numeric expressions that include both multiplication and division or both addition and subtraction. Remind your student that multiplication and division are of equal importance and evaluated in order from left to right. The same is true for addition and subtraction.

Evaluate each expression using the order of operations.

$$\mathbf{-168 \div 2^3 - 3^3 + 20}$$

$-168 \div \mathbf{2^3} - \mathbf{3^3} + 20$	Exponents
$-168 \div \mathbf{8} - \mathbf{27} + 20$	
$\mathbf{-168 \div 8} - 27 + 20$	Division
$\mathbf{-21} - 27 + 20$	
$\mathbf{-21 - 27} + 20$	Subtraction
$\mathbf{-48} + 20$	
$\mathbf{-48 + 20}$	Addition
$\mathbf{-28}$	

$$\mathbf{18 \div 2 \cdot 3^2}$$

$18 \div 2 \cdot \mathbf{3^2}$	Exponents
$18 \div 2 \cdot \mathbf{9}$	
$\mathbf{18 \div 2} \cdot 9$	Division
$\mathbf{9} \cdot 9$	
$\mathbf{9 \cdot 9}$	Multiplication
$\mathbf{81}$	

MYTH

"I learn best when the instruction matches my learning style."

If asked, some people will tell you they have a *learning style*—the expressed preference in learning by seeing images, hearing speech, seeing words, or being able to physically interact with material. Some people even believe that it is the teacher's job to present the information in accordance with that preference.

However, it turns out that the best scientific evidence available does not support learning styles. In other words, when an auditory learner receives instruction about content through a visual model, they do just as well as auditory learners who receive spoken information. Students may have a *preference* for visuals, writing, or sound, but sticking to their preference doesn't help them learn any better. It is far more important to ensure that the student is engaged in an interactive learning activity and to connect new information to the student's prior knowledge."

#mathmythbusted

In **Lesson 2: Introduction to Algebraic Expressions**, students develop their understanding of variables and algebraic expressions.

Variables and Algebraic Expressions

To **evaluate an algebraic expression** means to determine the value of the expression for a given value of each variable. When you evaluate an algebraic expression, you substitute the given value for the variables and then determine the value of the expression using order of operations.

h	$3h - 2$
4	$3(4) - 2 = 12 - 2 = 10$
–2	$3(-2) - 2 = -6 - 2 = -8$
$\frac{7}{3}$	$3\left(\frac{7}{3}\right) - 2 = 7 - 2 = 5$
5.1	$3(5.1) - 2 = 15.3 - 2 = 13.3$

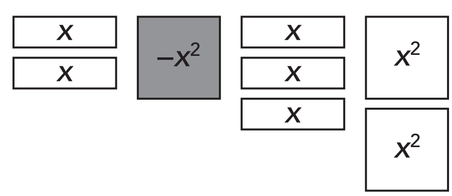
In **Lesson 3: Equivalent Expressions**, students continue their work with algebra tiles as a helpful tool to make sense of rewriting algebraic expressions.

Algebra Tiles

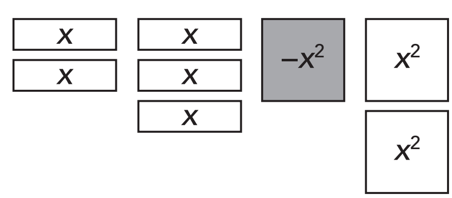
Your student will use algebra tiles and properties of numbers and operations to form **equivalent expressions**, just as they did in previous lessons with numeric expressions.

Consider the model. Write an addition expression that highlights the different tiles in the model.

$2x - x^2 + 3x + 2x^2$

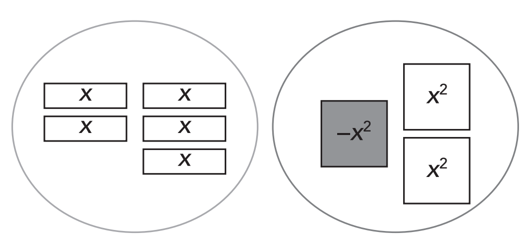


The tiles can be rearranged to combine all the like tiles.



There are now 2 terms in the expression: x and x^2

The new algebraic expression represented is
$5x + x^2$



TOPIC 2 Equations and Inequalities

In this topic, students use their understanding of expressions to develop an understanding of the equals sign as indicating a relationship, not as an operator. They learn that solving an equation means maintaining equality of the expressions on either side of the equals sign. Students use bar models to reason about solving one-step addition and multiplication equations. Students solve a variety of real-world problems by writing and solving one-step equations and inequalities. They recognize that solving inequalities is similar to solving equations, but there is a special case when multiplying and dividing both sides of an inequality by a negative number. Students then represent solutions to inequalities on number lines.

Where have we been?

Students have been representing and solving problems involving four operations since second grade. They have used drawings and equations with a symbol for the unknown number to represent the problem. They have determined unknown whole numbers in equations by relating whole numbers. The topic begins by asking students to set expressions equal to each other and then determine whether they are truly equal. Students will continue to use their knowledge of writing and evaluating expressions as they write and solve equations.

Where are we going?

In future courses, students will expand their ability to solve equations to two-step linear equations. They will expand their skills to include solving exponential, quadratic, polynomial, and trigonometric equations. All of this work is built upon the foundation of equivalent expressions that students begin to build in this topic.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Equations and inequalities can be tricky, and students are bound to make mistakes as they practice these concepts. Remember that making mistakes is an essential part of learning. Encourage your student to use prior knowledge and construct models to make sense of the mathematics they are learning. While you may be tempted to point out errors, it is more beneficial to ask clarifying questions to guide your student in the right direction. Help your student take ownership of their learning by asking them to analyze their work and self-correct. This will support your student as they build confidence as a mathematician.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does you answer make sense? How do you know?

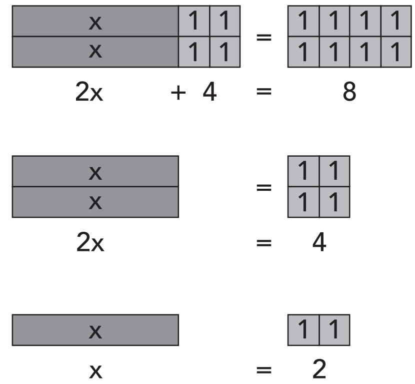
NEW KEY TERMS

- equation [ecuación]
- reflexive property of equality [propiedad reflexiva de la igualdad]
- solution [solución]
- addition property of equality [propiedad de suma de la igualdad]
- subtraction property of equality
- multiplication property of equality [propiedad de multiplicación de la igualdad]
- division property of equality [propiedad de división de la igualdad]
- symmetric property of equality [propiedad simétrica de la igualdad]
- zero property of multiplication [propiedad del cero de la multiplicación]
- identity property of multiplication [propiedad de identidad de la multiplicación]
- identity property of addition [propiedad de identidad de la suma]
- graph of an inequality [gráfica de una desigualdad]
- solution set of an inequality
- bar model [modelo de barras]

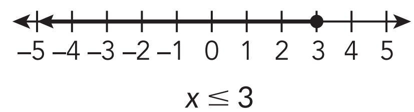
Where are we now?

A solution to an **equation** is any value for a variable that makes the equation true.

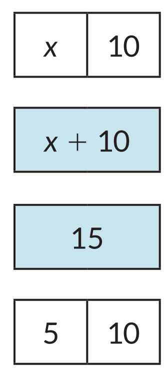
The solution to the equation $2x + 4 = 8$ is $x = 2$.



The **graph of an inequality** in one variable is the set of all points on a number line that make the inequality true.



A **bar model** uses rectangular bars to represent known and unknown quantities.
You can use a bar model to solve the equation $x + 10 = 15$.
The top bar can be split into two bars, x and 10. When this split happens in the bottom bar, with one bar containing 10, it shows that x is the same as 5, so $x = 5$.



In **Lesson 1: Reasoning with Equal Expressions**, students learn that an equation is a mathematical sentence created by equating two expressions.

Properties of Equality

The properties of equality are rules that allow you to maintain balance and rewrite equations.

Properties of equality	For all numbers *a*, *b*, and *c*
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \cdot c = b \cdot c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Symmetric property of equality	If $a = b$, then $b = a$.

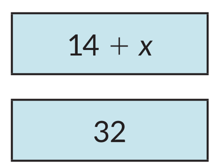
- one-step equation
- inverse operations [operación inversa]
- literal equation [ecuación literal]
- Inequality [desigualdad]
- solve an inequality [resolver una desigualdad]
- properties of inequalities [propiedades de las desigualdades]

Refer to the Math Glossary for definitions of the New Key Terms.

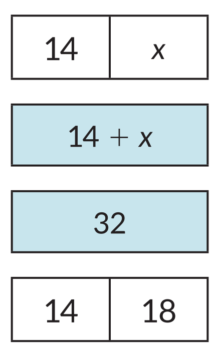
In **Lesson 2: Solving One-Step Addition Equations**, students use bar models to solve a variety of one-step addition equations.

Using Bar Models to Add

Consider the addition equation $14 + x = 32$.



This equation states that for some value of x, the expression $14 + x$ is equal to 32. In other words, some number added to 14 will equal 32. This can be represented using a bar model.



Just like with area models, bar models can be decomposed. The expression $14 + x$ can be decomposed into a part representing x and a part representing 14. The number 32 can be decomposed in a similar way: $32 = 14 + 18$. The bar model demonstrates that these two equations are equivalent.

$$14 + x = 32$$
$$14 + x = 14 + 18$$

Looking at the structure of the second equation, you can see that 18 is the value of x that makes this equation true.

In Lesson 3: Solving One-Step Multiplication Equations, students use bar models to reason about and solve a variety of one-step multiplication equations.

BY NC

MYTH

Just give me the rule. If I know the rule, then I understand the math.

Memorize the following rule: *All quars are elos*. Will you remember that rule tomorrow? Nope. Why not? Because it has no meaning. It isn't connected to anything you know. What if we change the rule to: *All squares are parallelograms*. How about now? Can you remember that? Of course you can because now it makes sense. Learning does not take place in a vacuum. It *must be* connected to what you already know. Otherwise, arbitrary rules will be forgotten.

#mathmythbusted

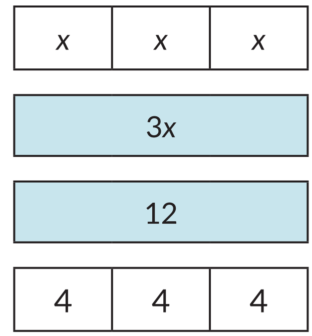
Using Bar Models to Multiply

Consider the multiplication equation $3x = 12$. This equation states that for some value of x, the expression $3x$ is equal to 12. This can be represented using a bar model.

You can decompose $3x$ by rewriting it as the equivalent expression $1x + 1x + 1x$, or $x + x + x$.

To maintain equivalence, decompose 12 in a similar way.

The bar model demonstrates that these two equations are equivalent.



$$3x = 12$$
$$x + x + x = 4 + 4 + 4$$

By examining the structure of the second equation, you can see that $x = 4$.

In Lesson 5: Solving Inequalities with Inverse Operations, students solve inequalities and graph the solutions on number lines.

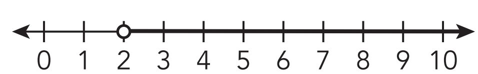
Inequalities

An **inequality** is any mathematical sentence that has an inequality symbol. To **solve an inequality** means to determine the solution set, which includes all values of the variable that make the inequality statement true.

Solve the inequality and graph the solution set on the number line.

$$13 < x + 11$$

The inequality can be read as 13 is less than x plus 11.

$13 < x + 11$	Given
$13 - 11 < x + 11 - 11$	Subtract 11 from both sides to create a zero pair.
$2 < x$ or $x > 2$	

2 is less than x.
x is greater than 2.

Your student will also learn that the inequality symbol will stay the same when adding, subtracting, multiplying, and dividing both sides of the inequality by a positive number. However, the inequality symbol reverses when multiplying or dividing both sides of an inequality by a negative number.

For example, solve the following inequalities using the **properties of inequalities**.

$$x + 4 \geq -9$$

$$x + 4 - 4 \geq -9 - 4$$

$$x \geq -13$$

The inequality symbol does not reverse since it involves only adding to or subtracting from both sides of the inequality.

$$\frac{x}{8} < -7$$

$$(8)\frac{x}{8} < -7(8)$$

$$x < -56$$

The inequality symbol does not reverse since it involves multiplying or dividing both sides by a positive number.

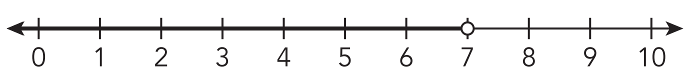
$$-3x > -21$$

$$\frac{-3x}{-3} < \frac{-21}{-3}$$

$$x < 7$$

The inequality symbol does reverse since it involves multiplying or dividing both sides by a negative number.

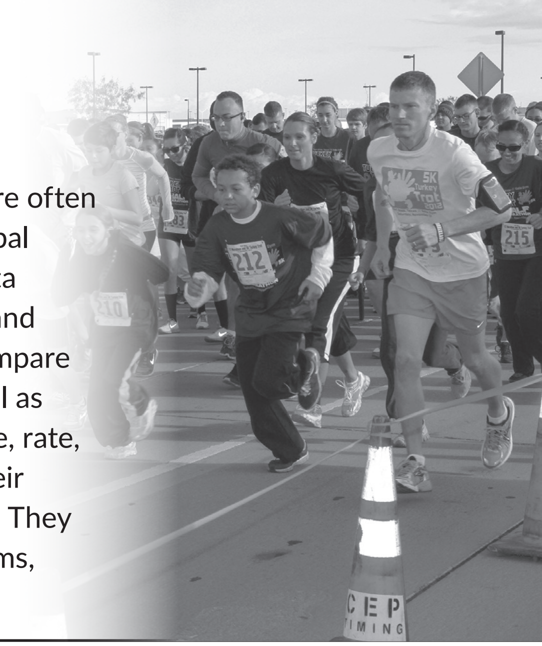
The solution to any inequality can be represented on a number line by a ray in which its starting point is an open or closed circle. For example, the solution $x < 7$ is represented by this number line. Notice that an open circle is used to represent that 7 is not included in the solution. If the inequality $x \leq 7$ were being represented, then a closed circle, or solid black dot, would be used to show that 7 is included in the solution.



Family Guide

MODULE 4 Determining Unknown Quantities Grade 6

TOPIC 3 Graphing Quantitative Relationships



Students learn that quantities can vary in relation to each other and are often classified as independent and dependent quantities. Students use verbal descriptions, tables, graphs, and equations to display and interpret data as well as to solve for unknown values. They analyze the advantages and limitations of each representation. Throughout the topic, students compare and contrast linear equations of the forms $y = kx$ and $y = x + b$ as well as their respective representations. They analyze three different distance, rate, and time scenarios and generalize the formula $d = rt$. Students use their knowledge of unit rates and unit conversions to solve these problems. They wrap up this topic by representing and solving a wide range of problems, including those with real-world scenarios.

Where have we been?

In Grade 5, students graphed and named ordered pairs in the first quadrant of the coordinate plane. In a previous topic, students graphed points in all four quadrants. In this topic, students graph and interpret scenarios that include all quadrants. This topic also connects to prior experiences with graphing equivalent ratios and writing inequalities with constraints.

Where are we going?

This topic provides the foundation for the formal study of independent and dependent variables. As students continue their mathematical journeys, they will be required to identify and solve for dependent and independent variables using a variety of tools.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to make connections between algebraic representations, tables, and graphs.

SOME THINGS TO LOOK FOR

Discuss graphs you see online, on television, and in print. Talk about what the graph is demonstrating and what two (or more) quantities it is comparing.

QUESTIONS TO ASK

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

NEW KEY TERMS

- dependent quantity [cantidad dependiente]
- independent quantity [cantidad independiente]
- independent variable [variable independiente]
- dependent variable [variable dependiente]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **dependent quantity** is the quantity that depends on another in a problem situation.

For example:
Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

number of gallons $\cdot \frac{\text{miles}}{\text{gallon}} =$ miles traveled

The dependent quantity is the total miles traveled. The number of miles traveled depends on the number of gallons of fuel.

The **independent quantity** is the quantity that the dependent quantity depends on.

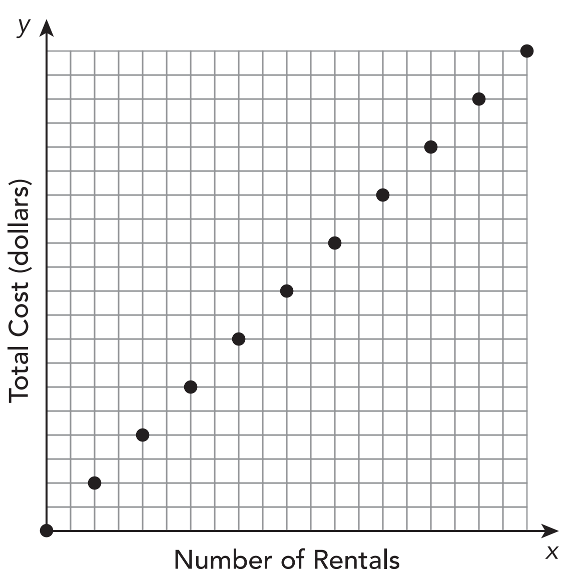
For example:
Max just got a new hybrid car that averages 51 miles to the gallon. How far does the car travel on 15 gallons of fuel?

number of gallons $\cdot \frac{\text{miles}}{\text{gallon}} =$ miles traveled

The independent quantity is the number of gallons. The other quantity (miles traveled) is dependent upon this quantity.

In **Lesson 1: Independent and Dependent Variables**, students will determine how one quantity depends on another using scenarios, equations, and graphs.

Independent and Dependent Variables



Suppose the video kiosk charges $2.00 for DVD and game rentals. How many DVDs and games can you rent for different amounts of money?

In this scenario, the **independent quantity** is the number of rentals, and the dependent quantity is the total cost in dollars. The equation that represents the scenario is $t = 2r$. The independent variable is r, which represents the number of DVD and games rented, and the dependent variable is t, which represents the total cost.

The independent quantity is plotted on the x-axis, and the **dependent quantity** is plotted on the y-axis.

In **Lesson 2: Using Graphs to Solve Problems**, students determine unknown values for a scenario and use the values to write a one-step multiplication equation to represent the scenario.

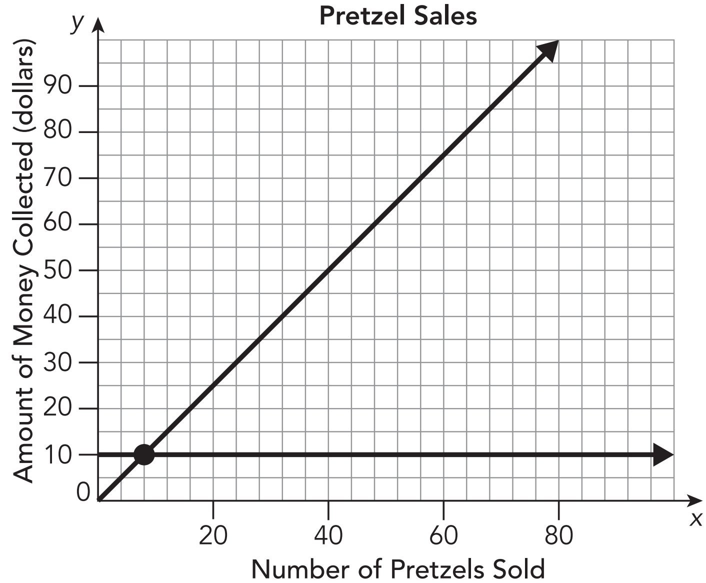
Using Graphs to Solve Problems

A graph can be used to determine how many pretzels Mario sold if he collected $10.

First, locate 10 on the y-axis and draw a horizontal line. This shows that $10 is the amount of money collected. The *x*-value of the point where your horizontal line intersects with the graph of $y = 1.25x$ is the number of pretzels sold for $10.

Remember, the solution to an equation is any value that makes the equation true. On the graph, a solution is any point on that graph.

The graph shows that Mario sold 8 pretzels and collected $10.



In **Lesson 3: Multiple Representations of Equations**, students analyze equations in a variety of forms—represented in tables, graphs, word problems, and algebraic equations.

Multiple Representations of Equations

Each representation shows the relationship between the distance of a train from the station and the time in minutes.

Equation

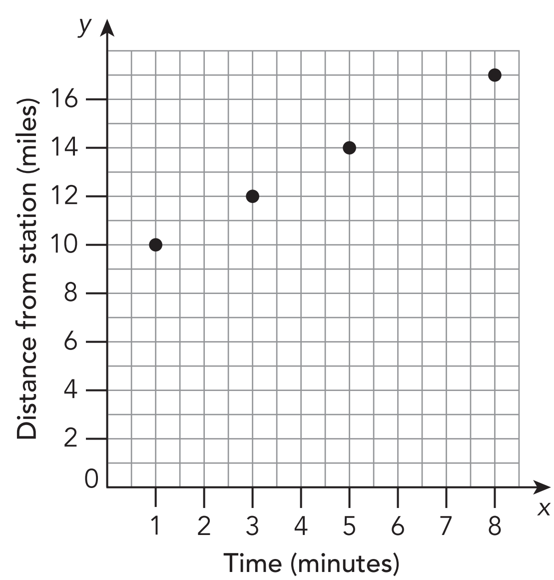
$$d = t + 9$$

t represents the time in minutes, and *d* represents the distance in miles.

Table

Time (minutes)	Distance (miles)
1	10
3	12
5	14
8	17

Graph



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MYTH

Myth: Memory is like an audio or video recording.

Let's play a game. Memorize the following list of words: *strawberry*, *grape*, *watermelon*, *banana*, *orange*, *peach*, *cherry*, *blueberry*, *raspberry*. Got it? Good. Some believe that the brain stores memories in pristine form. The idea is that memories last for a long time and don't change—like recordings. Without looking back at the original list, was *apple* on it?

If you answered "*yes*," then go back and look at the list. You'll see that *apple* does not appear, even though it seems like it should. In other words, memory is an active, reconstructive process that takes additional information, like the category of words (e.g., fruit), and makes assumptions about the stored information.

This simple demonstration suggests that memory is *not* like a recording. Instead, it is influenced by prior knowledge and decays over time. Therefore, students need to see and engage with the same information multiple times to minimize forgetting.

#mathmythbusted

In **Lesson 4: Relating Distance, Rate, and Time**; students analyze and solve problems about competing in triathlons to investigate the relationship $d = rt$.

Distance, Rate and Time

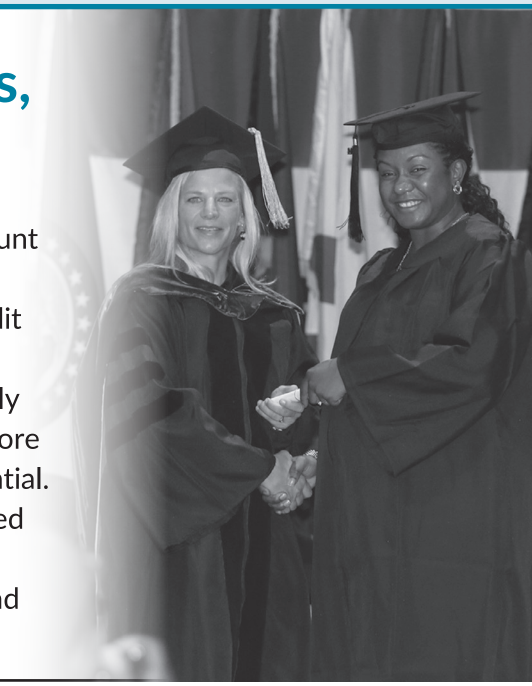
You can use the equation $d = rt$, where d represents the distance traveled, r represents the rate of the distance traveled to the time, and t represents the time, to solve a variety of real-world problems. This formula is an example of a real-world application of the equation $y = kx$.

For example, Mia drives 60 miles per hour for 2 hours. How far did she drive?

$D = rt$
$D = (60)(2) = 120$

Mia drove 120 miles.

TOPIC 4 Financial Literacy: Accounts, Credit, and Careers



Students begin this topic learning about checking accounts, including how to balance a checkbook register, and they compare checking account options from different financial institutions. Students also distinguish between debit cards and credit cards. Next, students learn what a credit report is, how a credit score is determined, and how borrowers and lenders use credit reports. Students investigate career choices, primarily from an educational and financial perspective. They discuss that the more education or training a person receives, the greater their earning potential. They take into account both the earning potential of having an increased level of education and the cost of student loans to acquire that level of education. Students end the topic by exploring different options to fund post-secondary education tuition.

Where have we been?

Students have been studying financial literacy since kindergarten. They know how to distinguish between spending and saving and distinguish deposits and withdrawals. Students balance a simple budget and identify advantages and disadvantages of different methods of payment, including checks, credit cards, debit cards, and electronic payments. This foundation is formalized in this course with studying checking accounts, debit cards and credit cards in more depth.

Where are we going?

The financial literacy skills learned in this course are skills that will impact all students in their adult lives. This topic is important for developing future financially responsible citizens but also for helping students start thinking about career choice, the education required for different careers, the potential cost for that post-secondary education, and various methods for paying for that education.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

Your student is learning about post-secondary options for college and careers. They consider different career choices, the education required, potential costs, and various ways to fund their choices.

QUESTIONS TO ASK

- What are some of the options available for post-secondary education? What are the advantages and disadvantages of each?
- What options are available to help offset the cost of tuition?
- What other factors might influence your decisions related to post-secondary education?

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NEW KEY TERMS

- checking account
- check [cheque]
- statement
- account balance
- deposit [depósito]
- withdrawal
- debit [débito]
- transfer [transferencia]
- overdraft
- Annual Percentage Yield (APY)
- debit card
- credit card
- interest [interés]
- interest rate
- credit report
- credit history [historial de crédito]
- credit score
- circle graph [gráfica circular]
- post-secondary education
- associate's degree
- undergraduate degree
- master's degree [título de maestría]
- vocational school [escuela vocacional]
- student loan
- tuition
- grant
- scholarship
- work-study program
- public school [escuela pública]
- private school [escuela privada]

Refer to the Math Glossary for definitions of the New Key Terms.

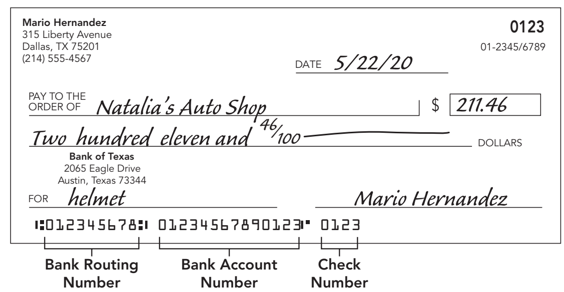
Where are we now?

Annual Percentage Yield (APY) is a percentage that is paid to customers based on the account balance in an account for a year.

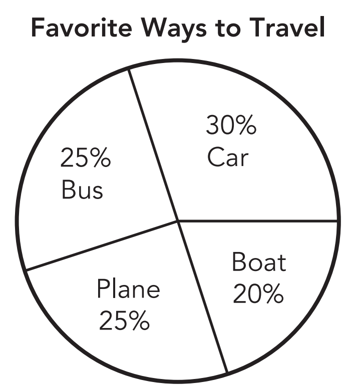
Ashley opens a checking account with an average monthly balance of \$600. The account has a 2.5% APY. $(600)(0.025) = 15$

Ashley will earn \$15 on this checking account over the course of the year.

A **check** is a written order to a bank to pay a specific amount of money to a person or company out of your checking account.



A **circle graph,** often called a *pie chart*, displays categorical data using sectors, or "wedges," of a circle.



In **Lesson 1: Checking Accounts,** students explore checking accounts and how to write checks.

Checking Accounts

Students learn about checking accounts, including how to balance a checkbook register, and they compare checking account options from different financial institutions.

Check	Date	Transaction Description	Payment/ Withdrawal	Deposit	Balance
103	Nov 1	Electric Company	$72.50		$1227.50
	Nov 1	ATM withdrawal	$80.00		
	Nov 2	ATM deposit		$50.00	

- A **withdrawal** occurs when money is taken out of the account, usually from a check, online payment, or Automated Teller Machine (ATM).
- A **deposit** is money put into the account. Deposits may come in the form of cash, checks, or transfers. Paychecks are often deposited directly into a checking account.
- The **account balance** is the amount of money in the account at a given time.

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In **Lesson 2: Debit Cards vs. Credit Cards**, students compare and contrast the key characteristics of debit cards and credit cards.

Debit Cards versus Credit Cards

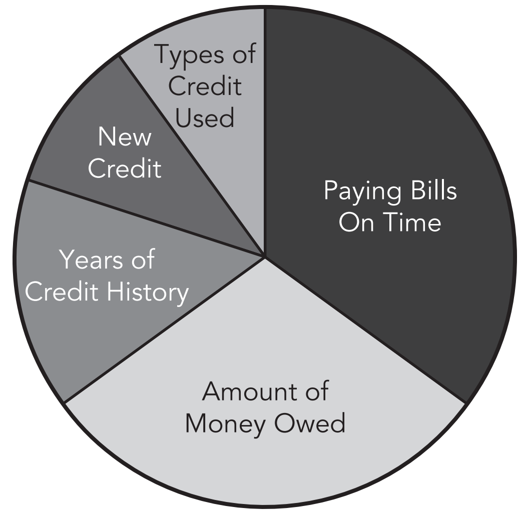
Debit Card	Credit Card
A debit card is a card that allows a bank's customer to make purchases using money from their account. • Issued by the bank • Generally given to all bank customers when they open a checking account • Funds are taken in full directly from the customer's checking account. • Protected by a security code (PIN, or Personal Identification Number) • Because money is immediately taken out, recovering funds due to fraud or theft may be difficult. • Spending is limited by the amount of money in your account.	A credit card is a card that allows a person to borrow a certain amount of money and pay the borrowed money back at a later time. • Customer applies for a credit card through a financial Company. • Money is loaned to the person to cover expenses. • A bill for all purchases charged is sent at the end of the month. • Additional money, based on a percentage of the amount charged, is added to the bill if not paid in full at the end of each month. • Protected by the customer's signature • Customers are protected from fraud or theft. • Often offer rewards for their use • Each card has a limit to the amount that a customer can borrow.

In **Lesson 3: Understanding Credit Reports**, students learn what a credit report is, how a credit score is determined, and how borrowers and lenders use credit reports.

Credit Reports

The circle graph provides a breakdown of the importance of the information in a credit report when determining a credit score.

Credit Score Factors



Paying bills on time and the amount of money owed are the top two factors that affect a credit score.

In **Lesson 4: Career Exploration**, students address career choice primarily from an educational and financial perspective.

Education and Careers

Students learn that in general, the more education or training you receive, the greater the financial benefit will be. A certain career may require one, two, four, or even eight years of education after graduating from high school.

Education Level Earned	Financial Benefit
Master's Degree	A person with a master's degree earns approximately 30% more annually than a person with only an undergraduate degree.
Undergraduate Degree	A person with an undergraduate degree earns 84% more over a lifetime than a person with just a high school diploma.
High School Graduate	A high school graduate earns \$350,000 more over a lifetime than a person without a high school diploma.

College degrees can provide a sense of accomplishment and allow the opportunity to pursue a career in a field of interest. Schooling provides large financial benefits as well.

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MYTH

Once I understand something, it has been learned.

Learning is tricky for three reasons. First, even when we learn something, we don't always recognize when that knowledge is useful. For example, you know there are four quarters in a dollar. But if someone asks you, "What is 75 times 2?" you might not immediately recognize that it is the same thing as having six quarters.

Second, when you learn something new, it's not as if the old way of thinking goes away. For example, some children think of north as straight ahead. But have you ever been following directions on your phone and made a wrong turn, only to catch yourself and think, "I know better than that!"

The final reason that learning is tricky is that it is balanced by a different mental process: forgetting. Even when you learn something (e.g., your locker combination), when you stop using it (e.g., you get a new locker), it becomes extremely hard to remember.

There should always be an asterisk next to the word when we say we learned* something.

#mathmythbusted

In **Lesson 5: Paying for College,** students explore methods to fund tuition. They research different scholarships and how to locate grant money.

Paying for College

Tuition is the fee paid to receive instruction at a school. Tuition may be paid in a variety of ways, through personal savings, student loans, grants, scholarships, or work-study programs.

Student Loans A **student loan** is money borrowed to pay for college or trade school.	**Grants** A **grant** is money awarded, or given, to a person from the government or school to help pay tuition.
Scholarships A **scholarship** is money awarded to a person from the college or a private institution to help a student pay for school. Scholarships may be academic, athletic, based on financial need, or awarded from private organizations for a variety of reasons.	**Work-study Program** A **work-study program** is a plan set up by the college that allows a student to work while in school to earn money.

GRADE 6

MODULE

5

Describing Variability of Quantities

Family Guide

MODULE 5 Describing Variability of Quantities — Grade 6

TOPIC 1 The Statistical Process

In this topic, students are introduced to the statistical problem-solving process: formulate questions, collect data, analyze data, and interpret the results. Students will use this process throughout their studies of statistics, increasing the complexity of each step of the process as they develop statistical literacy. Students use bar graphs to analyze and interpret survey data, the final steps of the statistical process. As students learn about and analyze dot plots, stem-and-leaf plots, and histograms, they practice using the four steps of the statistical process.

Where have we been?

Students have engaged informally in the statistical problem-solving process throughout previous courses. They have organized, represented, and interpreted categorical data. Students have created picture graphs and bar graphs, and they have made dot plots to display data.

Where are we going?

In future courses, students will use the data displays learned in this topic to compare data distributions. They will use statistical problem solving to investigate and draw inferences about populations. Students will move into comparing two variables with data.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can support your student's learning by approaching problems slowly. Students may observe a classmate learning things very quickly, and they can easily come to the conclusion that mathematics is about getting the right answer as quickly as possible. When this doesn't happen for them, future encounters with math can make students' really nervous, making problem solving more difficult and reinforcing students' views of themselves as "not good at math." Slowing down is not the ultimate cure for math difficulties. But, it's a good first step for children who are struggling. You can reinforce the view that learning with understanding takes time and that slow, deliberate work is the rule, not the exception.

QUESTIONS TO ASK

- What strategy are you using?
- What is another way to solve the problem?
- Can you draw a model?
- Can you come back to this problem after doing some other problems?

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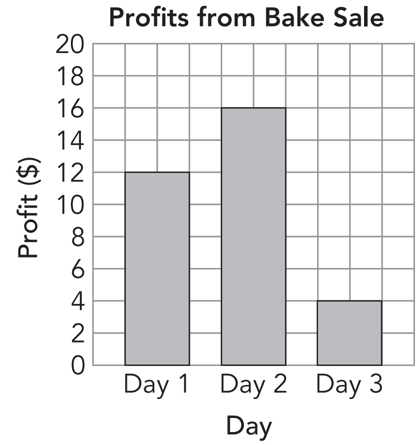
NEW KEY TERMS

- statistical question
- data [datos]
- variability [variabilidad]
- statistical process [proceso estadístico]
- categorical data [datos categóricos]
- quantitative data [datos cuantitativos]
- discrete data [datos discretos]
- continuous data [datos continuos]
- population [población]
- sample
- survey
- observational study [estudio observacional]
- experiment [experimento]
- bar graph [gráfica de barras]
- frequency [frecuencia]
- frequency table [tabla de frecuencia]
- mode [moda]
- relative frequency table [tabla de frecuencia relativa]
- dot plot
- distribution [distribución]
- symmetric distribution [distribución simétrica]
- skewed right distribution
- skewed left distribution

Where are we now?

A **bar graph** displays **categorical data** using either horizontal or vertical bars on a graph. The height or length of each bar indicates the value for that category.

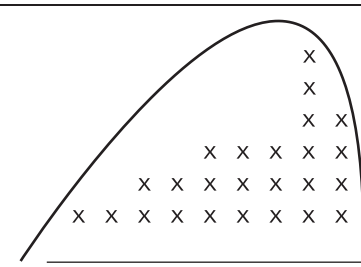


A **frequency table** is a table used to organize data according to how many times a data value occurs.

Number of Roller Coasters at Major Theme Parks

Number of Roller Coasters	Frequency (*f*)
7–9	6
10–12	4
13–15	7
16–18	2

In a **skewed left** distribution of data, the peak of the data is to the right side of the graph. There are only a few data points to the left side of the graph.



In **Lesson 1: Understanding the Statistical Process**, students are introduced to the statistical problem-solving process: create questions, collect data, analyze data, and interpret the results.

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- gaps
- peaks [picos]
- outliers
- stem-and-leaf plot
- key
- histogram [histograma]

Refer to the Math Glossary for definitions of New Key Terms.

Statistical Question

A **statistical question** is a question that has an answer based on data that vary.

Statistical Question

- What clubs do my classmates belong to?
- How many members do the clubs at my school have?

These examples are statistical questions because the answers will vary. Every student does not belong to the same clubs, and every club does not have the same number of members.

Not a Statistical Question

- What clubs am I in?
- How many students are in the chess club?

These examples are not statistical questions because each has only one answer; there is no variability in the answers to the questions.

Data Collection

The second step of the **statistical process** is to collect the data to answer the statistical question. Two types of data that can be collected are *categorical data* and *quantitative data.*

Categorical data, or *qualitative data*, are data for which each piece of data fits into exactly one of several different groups or categories.

Statistical Question:
What is the most popular favorite color in the school?
Sample answers:
Colors: blue, green, red, etc.

Quantitative data, or *numerical data*, are data for which each piece of data can be placed on a numerical scale and compared.

Statistical Question:
How many siblings do 6th graders have?
Sample answers:
Numbers: 0, 5, 3, 7, etc.

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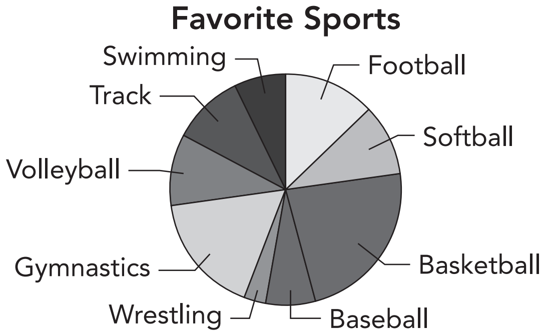
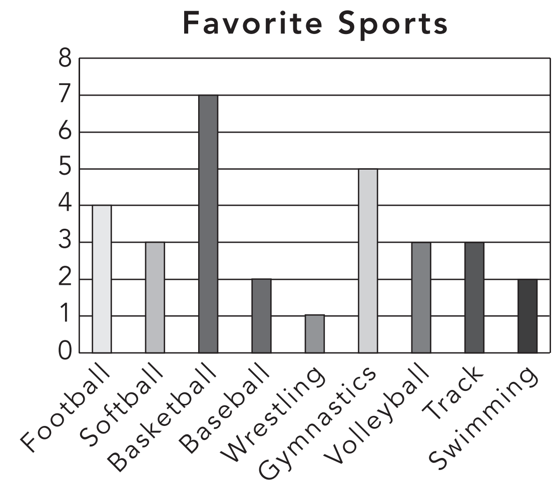
Analyze the Data and Interpret the Results

Part 3 of the statistical process is analyzing the collected data using different graphs. In this example, 30 students were surveyed on their favorite sport. The final step of the statistical process is interpreting the data, or drawing conclusions. By looking at both graphs you can conclude that basketball is the most popular sport among the **population** surveyed and that wrestling is the least popular sport.

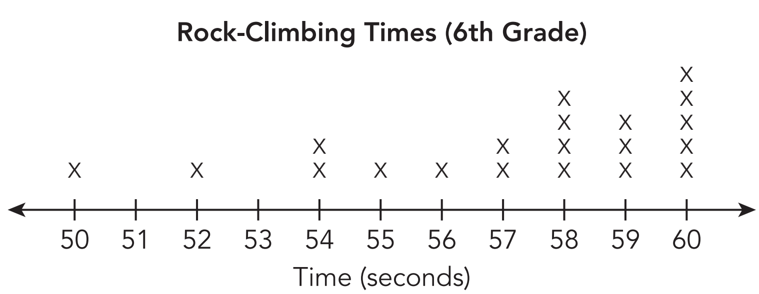




In **Lesson 2: Analyzing Numerical Data Displays** and **Lesson 3: Using Histograms to Display Data**, students interpret the numerical data displays of dot plots, stem-and-leaf plots, and histograms.

Graphs, Graphs, and More Graphs

A **dot plot** is a data display that shows discrete data on a number line with dots, Xs, or other symbols. Dot plots help organize and display a small number of data points.



Sixth-Grade Completion Times (seconds): 60, 50, 58, 59, 60, 54, 55, 58, 59, 60, 52, 54, 56, 57, 57, 58, 60, 60, 59, 58

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A **stem-and-leaf plot** is a table used to represent ordered data. Once the data is ordered, the 'stem' on the left displays the first digit or digits. The 'leaf' is on the right and displays the last digit.

Total Medals Won by Countries 2020 Summer Olympics

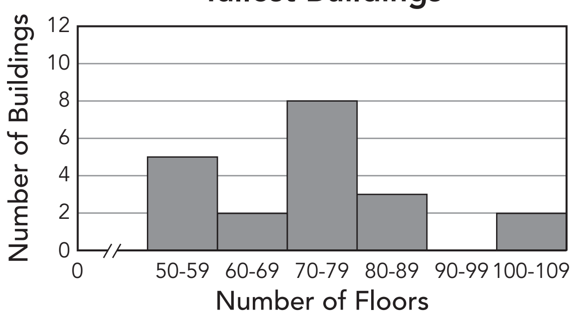
0	1 2 2 3 4 4 4 4 5 5 7 7 7 7 8 8 8 9 9 9
1	0 1 1 3 3 4 5 7 9
2	0 0 0 1 4
3	3 6 7
4	0 6
5	8
6	4
7	1
8	9
9	
10	
11	3

Key: 4|6 = 46 medals won

Total Medals Won by Countries 2020 Summer Olympics: 1, 2, 2, 3, 4, 4, 4, 4, 5, 5, 7, 7, 7, 7, 8, 8, 8, 9, 9, 9, 10, 11, 11, 13, 13, 14, 15, 17, 19, 20, 20, 20, 21, 24, 33, 36, 37, 40, 46, 58, 64, 71, 89, 113.

A **histogram** is a graph that displays quantitative or numerical data using vertical bars. The width of a bar represents an interval of data and is called a *range*. The height of the bar indicates the **frequency**, or the number of data values included in any given range.

Number of Floors in New York's Tallest Buildings



Number of Floors in New York's Tallest Buildings

Number of Floors	Number of Buildings
50–59	5
60–69	2
70–79	8
80–89	3
90–99	0
100–109	2

MYTH

Faster = smarter.

In most cases, speed has nothing to do with how smart you are. Why is that? Because it largely depends on how familiar you are with a topic. For example, a bike mechanic can look at a bike for about eight seconds and tell you details about the bike that you probably didn't even notice (e.g., the front tire is on backward). Is that person smart? Sure! Suppose, instead, you show the same bike mechanic a car. Will they be able to report the same amount of detail as they did for the bike? No!

It's easy to confuse speed with understanding. Speed is associated with the memorization of facts. Understanding, on the other hand, is a methodical, time-consuming process. Understanding is the result of asking lots of questions and seeing connections between different ideas. Many mathematicians who won the Fields Medal (i.e., the Nobel prize for mathematics) describe themselves as extremely slow thinkers. That's because mathematical thinking requires understanding over memorization.

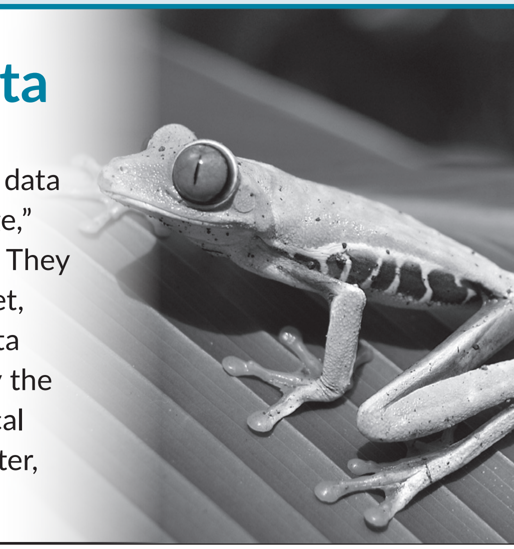
#mathmythbusted

Family Guide

MODULE 5 Describing Variability of Quantities

Grade 6

TOPIC 2 Numerical Summaries of Data



In this topic, students learn about measures of center and measures of variability, as well as when each measure is most appropriate for a given data set. Students may have an informal or intuitive understanding of "average," but this topic formalizes the ideas of the mean and median of a data set. They learn that the mean is the arithmetic average of the numbers in a data set, and that it can be interpreted as a fair share or the balance point of a data set. From there, students learn about measures of variability, specifically the interquartile range (IQR). Students summarize quantitative and categorical data with numerical and graphical summaries to describe the shape, center, and spread of the data distribution.

Where have we been?

In previous grades, students determined which value in a data set occurred the most; this measure is the mode. Students build on this knowledge as they learn about other measures of center: the median and the mean. When students learned about division, they created equal groups, which is a similar construct to the interpretation of the mean as a "fair share."

Where are we going?

This topic provides students with the building blocks of numerical data analysis: calculating measures of center and measures of variability to describe data. Students will continue to use these computations, and the reasoning behind them, as they compare data distributions and when they are introduced to mean absolute deviation as a measure of variability.

TALKING POINTS

DISCUSS WITH YOUR STUDENT

You can support your student's learning by asking questions about the work they do in class or at home. Your student is learning about the process of framing questions about data and representing data numerically.

QUESTIONS TO ASK

- What type of data display is the most appropriate for this set of data?
- Which measures of center are most useful when analyzing this set of data?
- Which do the measures of variability tell you about this set of data?

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NEW KEY TERMS

- measure of center
- mode [moda]
- median [mediana]
- balance point
- mean [media]
- measure of variation [medida de variabilidad]
- range [rango]
- quartile [cuartil]
- interquartile range (IQR) [rango intercuartílico]
- box plot
- frequency [frecuencia]
- double bar graph [gráfica de barras dobles]
- stacked bar graph
- percent bar graph [gráfica de barra de porcentaje]

Refer to the Math Glossary for definitions of the New Key Terms.

Where are we now?

A **box plot**, or just box plot, is a graph that displays the five-number summary of a data set: the **median**, the upper and lower **quartiles** (Q1 and Q3), and the minimum and maximum values.

Data: 32, 35, 35, 53, 55, 60, 60, 61, 61, 74, 74

Minimum = 32

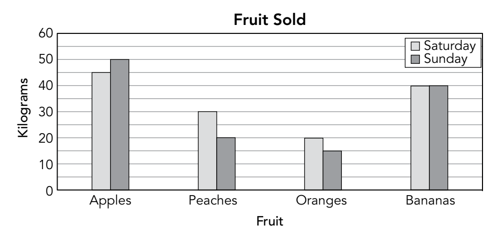
Q1 = 35

Median = 60

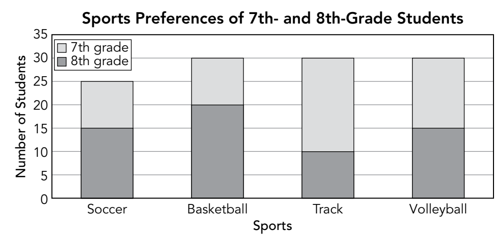
Q3 = 61

Maximum = 74

A **double bar graph** is used when each category contains two different data sets. The bars may be vertical or horizontal.



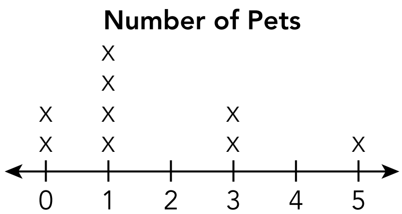
A **stacked bar graph** is a graph that stacks the frequencies of two different groups for a given category on top of one another so that you can compare the parts to the whole.



In **Lesson 1: Analyzing Data Using Measures of Center**, students learn about measures of center which tell you how the data values are clustered, or where the "center" of a graph of the data is located. There are three measures that describe how a data set is centered: the *mean*, the *median*, and the *mode*.

Mean

The **mean** is the arithmetic average of the numbers in a data set. It is based on leveling off or creating fair shares. In the example below there is a varying number of pets in the data set. The average number of pets is $1\frac{2}{3}$ pets.

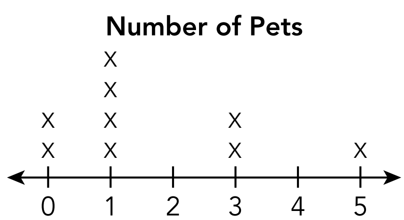


$$\text{Mean} = \frac{0 + 0 + 1 + 1 + 1 + 1 + 3 + 3 + 5}{9}$$
$$= \frac{15}{9} = 1\frac{2}{3} \text{ pets}$$

Nine people were surveyed. There are two people that have no pets, four people that have 1 pet, two people that have 3 pets, and one person that has 5 pets.

Median

The **median** is the middle number in a data set when the values are placed in order from least to greatest or greatest to least. When a data set has an odd number of data values, you can determine which number is exactly in the middle of the data set. If there is an even number of data values, the median is calculated by adding the two middle numbers and dividing by 2.



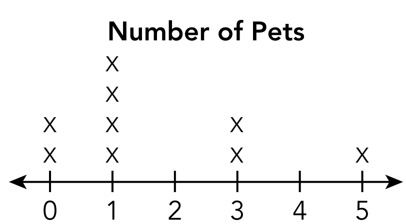
0, 0, 1, 1, 1, 1, 3, 3, 5

median

The median number of pets is one pet.

Mode

The mode is the data value or values that occur most frequently in a data set. A data set can have more than one mode.



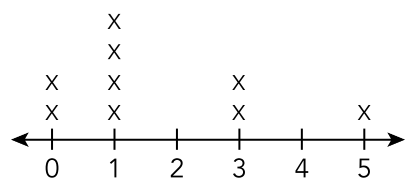
0, 0, 1, 1, 1, 1, 3, 3, 5
The mode of the data is 1.

In **Lesson 2: Displaying the Five-Number Summary**, students learn about the measures of variation which describes the spread of data values. The measures that describe the spread of data values are the *range*, *quartiles*, and the *interquartile range* (IQR).

Range

One measure of variation is the *range*. The **range** is the difference between the greatest and least values of a data set.

Number of Pets



0, 0, 1, 1, 1, 1, 3, 3, 5

$5 - 0 = 5$

The range of the data is 5.

Interquartile Range (IQR)

Your student will learn that a data set can be split into four parts called **quartiles** to describe the spread of the data values.

To summarize and describe the spread of the data values, you can use the five-number summary. The five-number summary includes these five values from a data set:

- Minimum: the least value in the data set
- Q1: the first quartile
- Median: the middle value in the data set
- Q3: the third quartile
- Maximum: the greatest value in a data set

first quartile (Q1) ↓ third quartile (Q3) ↓

Data: 32, 35, 35, 53, 55, 60, 60, 61, 61, 74, 74

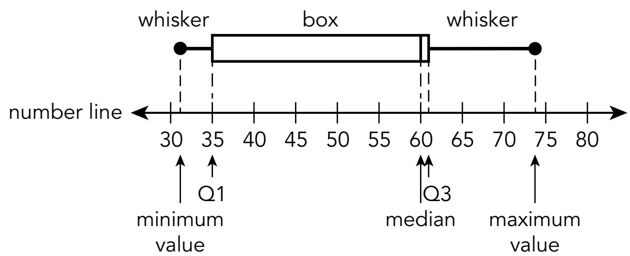
↑ second quartile/median (Q2)

- Minimum: 32
- Q1: 35
- Median: 60
- Q3: 61
- Maximum: 74

The **interquartile range** (IQR), is the difference between the third quartile, Q3, and the first quartile, Q1. The IQR indicates the range of the middle 50 percent of the data.

$$\text{IQR} = \text{Q3} - \text{Q1}$$
$$\text{IQR} = 61 - 35$$
$$\text{IQR} = 26$$

The five-number summary of a data set: the median, the upper and lower quartiles (Q1 and Q3), and the minimum and maximum values can be displayed on a **box plot**.



MYTH

Myth: "If I can get the right answer, then I should not have to explain why."

Sometimes you get the right answer for the wrong reasons. Suppose a student is asked, "What is 4 divided by 2?" and she confidently answers "2!" If she does not explain any further, then it might be assumed that she understands how to divide whole numbers. But, what if she used the following rule to solve that problem? "Subtract 2 from 4 one time." Even though she gave the right answer, she has an incomplete understanding of division.

However, if she is asked to explain her reasoning, either by drawing a picture, creating a model, or giving a different example, the teacher has a chance to remediate her flawed understanding. If teachers aren't exposed to their students' reasoning for both right and wrong answers, then they won't know about or be able to address misconceptions. This is important, because mathematics is cumulative in the sense that new lessons build upon previous understandings.

You should ask your student to explain their thinking, when possible, even if you don't know whether the explanation is correct. When children (and adults!) explain something to someone else, it helps them learn. Just the process of trying to explain is helpful.

#mathmythbusted

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